Geometry Epitomiz'd:

BEING

A Compendious Collection of the most useful Propositions in the First, Third, Fourth, Fifth and Sixth Books of *EUCLID*.

Together

With their Uses, in several practical Parts of the Mathematicks.

ALSO,

Euclid's second Book and Doctrine of Proportion, Alegebraically demonstrated. With some of the most useful Problems required in Practise.

By *WILLIAM ALINGHAM.*

LONDON,
Printed by *J. M.* and *B. B.* and are to be sold by the Author, over against the *Rummour Tavern* in *Channel Row*, Westminster, 1695.

Licens'd,

April the 12*th.* 1695.

D. Poplar.

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TO THE

Ingenious and my worthy Friend,

William Lownds

OF

WINSLO

IN

Buckinghamſhire, *Eſq;*

SIR,

AS *Geometry* has been a Science admir'd, and thought moſt uſeful by Ingenious Perſons in all Ages, and being fully ſatisfied that

your Curiosity has led you to the Knowledg of the highest Misteries in this *Science*.

I have from hence allow'd my self to hope for your Protection of this small Treatise, and do humbly beg it, as a favour more to be added, to those already you have been pleas'd to Confer upon

Sir,

Your Oblidged and

Humble Servant

Will. Alingham.

THE

PREFACE.

THE *World has been ſo hu-mour'd with* Preface *to a Book, like a Maſter of the Ceremo-nies to introduce it, that a Work though never ſo valuable appearing without it, will be rather redicul'd than read; that therefore ſo uſefull a Subject as the following Sheets treat off, ſhould not be ſo ſerv'd: I have per-fix'd theſe few lines to inform the* Rea-der.

That he is here preſented with a ſmall peece of Geometry, *a Science*

ſo

so universally useful, that it needs no Rhetorick to recommend it, the great and continual usefullness, with the no less pleasure that attend its knowledg, are sufficient motives to engage one in the study of it; and therefore to say much of the Excellency of this Science will be needless, when we consider that all Fabricks from the most magnificent Structure, to the meanest Cottage, owe their contrivance to it: For 'tis by this Art the Country Gentleman may Survey his Lands, measure his Timber, and contrive his Buildings, Gardens, Fountains, Aquaducts, &c. And if he understood it aright, would not repent Exchanging many of his beloved Pastimes, for this greater and more usefull Recreation.

The

The great Delight and Pleasure also of this Study is not less evident to those who are not sworn Enemies to the Art of thinking, but will allow an exercise of the Mind join'd with that of the Body to be the more noble sort of Recreation, for in the consideration of the simplest parts of body, viz. Points, Lines *and* Surfaces, *variety of delightful Properties are discovered, which surprize our mind with unexpected truth and conviction, and layeth a foundation to many noble and useful Rules of Practice, not only in several parts of the Mathematicks, but also in many instances of common life.*

In short, time would fail to enumerate the vast advantages that attend this Excellent and Noble Science to Persons in all Stations, and of all

Pro-

Professions whatsoever; insomuch that no person can be said to be compleatly accomplisht, without some competent knowledge herein.

But no longer to write in Praise of Hercules, *or commend that which none but inconsiderate persons did ever discommend, let me come to speake a little of the following Treatise, which I have presum'd to Entitle* Geometry Epitomiz'd, *though it be something to comprehensive for the nature of the Work, since I meddle not with* Solids *which is a great part of* Geometry. *My design (as the Title informs) being only a compendious contraction of the First, Third, Fourth, Fifth and Sixth Books of* Euclid, *which treat only of the first principles of* Geometry.

In the composing of which I *have carefully endeavour'd to rank the propositions in such order, as that they may depend upon each other, and from them have rightly drawn their principal* Scholions *and* Corollaryes, *so that I hope the whole undertaking is so perform'd, that the impartial* Reader *will find the Title made good in every respect.*

Yet if in what is hereafter delivered, any Mistake be committed, or some things not so clearly exprest as they might have been, I desire the courteous Reader *to consider, that the Press, and our Natures, are the causes of such Effects, Faults ever attending the former, and frailty the later; and therefore that as I doubt not but it will meet with the common fate of Books, to wit,*

wit, Cavilers, *so I also hope that there are others that will kindly accept of what is here offered, and freely pardon the Errors, remembring that they are humane.*

ERRATA.

Page 15. *l.* 3. *for* a Proportion *r.* the same Proportion.

p. 112. *in the Cut of the two single Lines,* a b *should have been the longest.*

Notes Explain'd.

= *Equal to, as* a = b.	is thus read	a *is equal to* b.
+ *More, as* a + b.		a *more* b, *or the Sum of* a *and* b.
— *Less, as* a — b.		a *less* b, *or the remainder after* b *is taken from* a.
⊏ *Greater than, as* a ⊏ b.		a *is greater than* b.
⊐ *Lesser than, as* a ⊐ b.		a *is lesser than* b.
x *Multiplied by, as* a x b.		a *multiplied by* b. *the same is denoted by the conjuction of Letters, thus* a b.
∠ *An Angle, as* ∠ a.		*The Angle* a.
∠s		*Angles.*
△ *Triangles, as* △ a b c.		*The Triangle* a b c.
√ *Root, as* √ a.		*The square Root of* a.

These four Points (: :) placed betwixt 4 *Quantities* is the Note of *Proportion*, as if the 4 *Quantities* stand in this order. $a : b :: c : d$, they are thus to be read, as *a* to *b*, so is *c* to *d*.

This Note ($\div\!\!:$) before any parcel of *Quantities*, shews them to be in *continual proportion*, as $\div\!\!:\ a\ b\ c\ d$, *&c.* are to be read as *a* to *b*, so is *b* to *c*, so is *c* to *d*, &c.

$\frac{a}{b}$ Two *Letters* or *Quantities* placed in this manner, notes Division, and signifies that *a* must be divided by *b*.

Lastly, $a \times \overline{b + c}$ denotes that *a* is to be Multiplied by the sum of *b* and *c*, understand the like by the difference of two *Quantities*.

Of

OF GEOMETRY.

GEometry, as to its *Name*, ſignifies properly no more than the *Meaſuring of the Earth*; but in general it is defined to be the *Science* of *Magnitude*, or *continued Quantity*, whoſe parts, though never ſo vaſt or remote, are by its own *Principle* and *Demonſtration* underſtood, and exactly meaſured, ſo that it hath the whole *Univerſe* for its *Subject*, and is indeed the moſt pure and noble of all the *Mathematick Sciences*.

It is principally divided into two parts, *viz. Speculative* and *Practick*. *Speculative*, is that which is employed about demonſtrating the properties of *Magnitudes*; the *Practick* is that which teacheth how to Deſcribe and meaſure thoſe *Magnitudes*, and as the firſt has his *Theorems*, ſo the latter hath his *Problems*.

Of this *Science* therefore, I ſhall lay down in the firſt place ſuch *Definitions* and *Principles*, as ought duely to be weighed and conſidered by the *Reader*, before he paſs to the *Demonſtration* and *Practice* of that which follows.

But before he enter on the *Definitions*, it may not be amiſs to inform him of the kinds of *Magnitude*, which are principally *Three*, to wit, *Length*, *Breadth* and *Thickneſs*, or a *Line*, a *Superfice* and a *Solid*; a *Line* being generated by the motion of a *Point*; a *Superficies* by the motion of a *Line*, and a *Solid* is produced by the motion of a *Superficies*, and which way ſoever a *Solid* is moved, it ſtill produceth but a *Solid*. Now what a *Point*, a *Line* and *Superficies* are, the following *Definitions* will declare unto you.

Definitions.

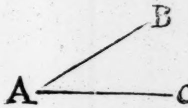
1. A *Point* is that which hath no part, as the prick *A*.

2. A *ſtraight* or *right line*, is the *ſhorteſt diſtance* betwixt two points, as the line *A B*.

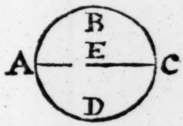
3. A *Superficies* is that which hath *length* and *breadth*, but no *thickneſs*.

4. A *plain Superficies* is the *ſhorteſt Space* that can be deſcribed, between one or more *right lines*, and is bounded by them.

5. A *plain* or *right lin'd Angle* is made by the *meeting* of two *right lines* in a *point*, as the *Angle A* is formed by the *right lines A B*, *A C*. An *Angle is commonly denoted by the middlemoſt of three Letters, as the Angle B A C denotes the Angle A.*



6. A *Circle* is a *plain Figure*, contained under one *line*, called a *Periphery* or *Circumference*; unto which from one point within the *Figure*, all lines drawn are equal, and theſe lines are called *Radius*'s or *Semidiameters*, as the Figure *A B C D*.

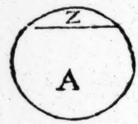


7. And that *point* in the middle of the figure noted with *E*, is called the *Center*.

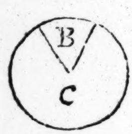
8. The *Diameter* of a *Circle*, is a *right line* paſſing through the *Center*, and is bounded on both ſides by the *circumference*, dividing the *Circle* into two equal parts, as the line *A C*.

9. A *Semicircle*, is that *figure* contained betwixt the *Diameter* and that part of the *Circumference* cut off by it, as *A B C*.

10. A *Segment* of a *Circle* is a part cut off, by a *right line* (leſs than the *Diameter*) drawn within the *Circle*, and the *Segment* may be either greater or leſſer than a *Semicircle*, as the figure *A* or Z.



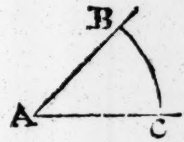
11. A *Sector* is formed by part of a *Circle*, and two *ſemidiameters* drawn from the *Center* to the *Circumference*, as the Figures C and *B*, the one being greater, the other leſs than a *Semicircle*.



12. Every *Circle* whether great or ſmall, is ſuppoſed to be divided into 360 equal parts, called *Degrees*, and each *Degree* into 60 *Minutes*, and every *Minute* into 60 *Seconds*, and ſo to *Thirds*, *Fourths*, &c. The reaſon of dividing it into 360 parts, is, becauſe this number will admit of moſt equal Diviſions, as 2. 3. 4. 5. 6. *&c.* And therefore the beſt way to divide a *Circle* as aforeſaid, is firſt to divide it into 4 equal parts, then one of theſe into 3 equal parts, next one of theſe into 6 equal parts, and laſtly one of theſe 6, into 5 equal parts, one of which will be a *Degree*, or the $\frac{1}{360}$ of the whole *Circle*.

13. The

13. The *measure* of an *Angle* is an *Arch* of a Circle, described from the angular *point*, and lies intercepted betwixt the sides, as the *Arch* *B C*, and so many *Degrees* and *Minutes* as are in the *Arch* *B C*, so much is said to be the measure of it, or of the *Angle* *A*.

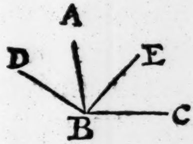


14. A *Plain*, or *right-lin'd Angle*, may be either *Right* or *Oblique*.

15. A *right Angle*, is that which hath an *Arch* of 90 Degrees for its measure, as the Angle *A B C*.

16. An *Oblique Angle*, is either *Acute* or *Obtuse*.

17. An *Acute Angle*, is that which is less than a *right Angle*, or 90 Degrees, as the Angle *E B C*.

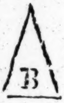


18. An *Obtuse Angle* is that which is greater than a *right Angle*, as the *Angle*, *D B C*.

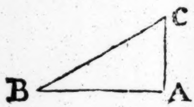
19. *Right lined Figures* are *such* as are contained under *straight lines*.

20. Of *three ſided Figures*, that is, an *Equilaterial Triangle*, which hath the three ſides equal one to the other, as the Figure *A*.

21. An *Iſoſceles Triangle*, hath onely two of its ſides equal as the Figure *B*.



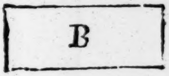
22. *Triangles* have alſo received Names from the *qualities* of their Angles, as if the Triangle have one *right Angle*; it is called a *right angled Triangle*, as the Figure *B A C*; in which *B A* is termed the *baſe*, *C A* the *Perpendicular*, and the ſide *B C* that ſubtends the *right Angle* the *Hypotenuſe*.



23. If a *Triangle* have no right Angle in it, 'tis termed an oblique *Angled Triangle*; and if ſuch Triangle have one obtuſe *Angle*, 'tis called an obtuſe *Angled Triangle*: But if no obtuſe or *right Angle*, then an accute *Angled Triangle*.

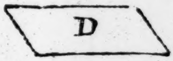
24. Of *Figures* that have four ſides, that is a *Square* which hath all its Sides equal, and Angles right, as the Figure *A*.

25. A *rectangle* or *long Square*, hath all four Angles right and oppoſite ſides equal as the Figure *B*,



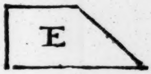
26. A *Rhombus* hath all its Sides equal, and oppoſite Angles equal, but none of them right, as the Figure *C*.

27. A *Rhomboids* is that whoſe oppoſite ſides are equal, as alſo the oppoſite Angles, but none of them right as the Figure *D*.



28. A *Parallelogram* is a *four ſided Figure*, whoſe oppoſite ſides are *parrallel*, as the Figures *A* and *B* in the 24 and 25 *Definitions.*

29. All other *Figures* having four ſides, are called *Trapezia*'s, as the Figure *E*.

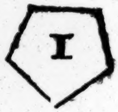


30. *Figures* that have above four ſides, and thoſe unequal, are called *many ſided* or *irregular poligons*, as the Figure G.

31. If *figures* have above four ſides, and thoſe equal, they are called *regular Poli-*

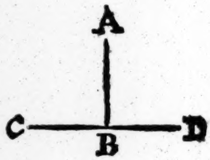
gons,

gons, as the Figure *I* of 5 equal ſides, is termed a *Pentagone*.

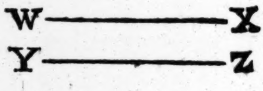


32. If a *figure* have ſix equal ſides, 'tis named a *Hexagon* or *Polygone* of ſix equal ſides, if ſeven, a *Polygone* of ſeven equal ſides, *&c.*

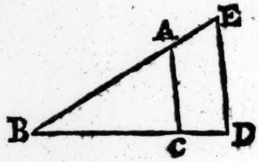
33. If a *ſtraight line A B* fall upon another *ſtraight line C D*, and make the Angles *A B C*, *A B D* equal to each other, then thoſe equal Angles are both *right Angles*, and the right line *A B* is termed a *perpendicular* to *C D*.



34. *Parrallel* or *Equidiſtant right lines* are ſuch, which if infinitely extended would never meet, ſuch are the *lines W X. Y Z.*

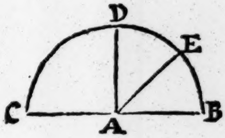


35. *Simillar* or *Equiangled Figures* are thoſe which have the ſeveral Angles of the one, equal ſeverally to the reſpective Angles of the other, and the ſides that form thoſe equal Angles proportional, as in the two triangles *B A C*, *B E D*, the



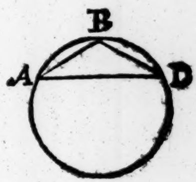
the Angle at *B* is common to both Triangles, the Angle at *E* in the greater, is equal to the Angle at *A* in the lesser, and the Angle *D* in the greater, is equal to the Angle C in the lesser; also *B E* : *E D* :: *B A* : *A C*.

36. *Complements* of *Angles* or *Arches* are so called in reference to a *Quadrant* or *Semicircle*, as the Angle *E A B* is the complement of the Angle *E A D* to a *Quadrant*, but of the Angle *E A C* to a *Semicircle*.



37. The *Diagonal* of any Figure is a right *line drawn* from the opposite Angles thereof, as in the *Trapezia* *B C D* the diagonal line is *B D*.

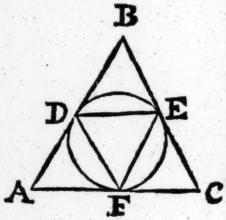
38. If a *point B* be taken in the *circumference* of a *Circle*, and from it *right lines B A*, *B D* drawn to the ends of the right line *A D*, which cuts off a *Segment*, then the Angle *A B D* contained under the adjoined lines *B A*, *B D* is said to be an *Angle* in a *Segment*.



39. *Segments* are said to be *Similar*, when the Angles so inscrib'd are equal.

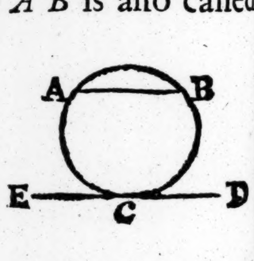
40. A

40. A *right lined Figure*, is ſaid to be inſcribed in a *right lined Figure*, when all the *Angles* of the one touch all the ſides of the other, as the *triangle D E F* is inſcrib'd in the *triangle A B C*, alſo the ſaid *triangle* is inſcribed in the *Circle D E F*, for the *periphery* thereof touches all *Angles* of the *Figure*.



41. A *right lined Figure* is ſaid to be deſcribed about a *right lined Figure*, when every ſide of the *circumſcribing Figure* touches every one of the Angles of the Figure, about which it is *circumſcribed*, as the triangle *ABC circumſcribes* the triangle *D E F*, alſo the Circle *D E F* is *deſcribed* about the triangle *D E F*.

42. A *right line* is ſaid to be *applied* in a Circle, when the ends thereof fall upon the *circumference*, as the line *A B* is applyed in the *Circle A B C*; the line *A B* is alſo called an *Inſcript*, and when ſo applyed, is ſaid to *cut* the Circle.

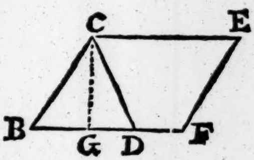


43. A *right line* is ſaid to touch a Circle when touching the ſame, and being produ-

ceth it cutteth it not, ſo the right line *E D* toucheth the Circle *A B C.*

44. In a Circle *CGDH*, right lines *CD*, *G H* are ſaid to be *equally diſtant* from the center *B*, when *perpendiculars B A*, *B E* drawn from the center *B* to them are equal, and *that line I K* is ſaid to be *fartheſt diſtant* from it, on which the longer *perpendicular B F* falleth.

45. The *Height* of any *Figure*, is a line drawn from the top of it, *perpendicular* to the baſe, as *C G* is the height or true breadth of the *Rhomboides B C E F*, it is alſo the height of the Triangle *B C D.*



What Proportion *is.*

46. *Proportion*, or *Reaſon*, is the *habitude* or *relation* that one *Magnitude* hath to another, according to *Quantity*: As if 7 be compar'd to 3, then the reſpect that the *quantity* of the *Magnitude* 7, has to the *quantity* of the *Magnitude* 3, is the *Reaſon* or *Habitude* of 7 to 3. In every *Ratio* obſerve, That that *Quantity* which hath reference, or is referr'd to another,

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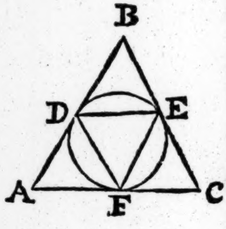
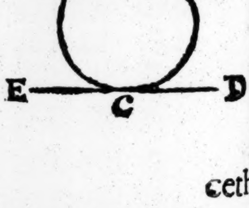


41. A *right lined Figure* is ſaid to be deſcribed about a *right lined Figure*, when every ſide of the *circumſcribing Figure* touches every one of the Angles of the Figure, about which it is *circumſcribed*, as the triangle *ABC* *circumſcribes* the triangle *D E F*, alſo the Circle *D E F* is *deſcribed* about the triangle *D E F*.

42. A *right line* is ſaid to be *applied* in a Circle, when the ends thereof fall upon the *circumference*, as the line *A B* is applyed in the *Circle A B C*; the line *A B* is alſo called an *Inſcript*, and when ſo applyed, is ſaid to *cut* the Circle.



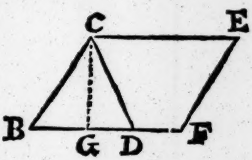
43. A *right line* is ſaid to touch a Circle when touching the ſame, and being produ-

ceth

ceth it cutteth it not, ſo the right line *E D* toucheth the Circle *A B C*.

44. In a Circle *CGDH*, right lines *CD*, *G H* are ſaid to be *equally diſtant* from the center *B*, when *perpendiculars B A*, *B E* drawn from the center *B* to them are equal, and *that line I K* is ſaid to be *fartheſt diſtant* from it, on which the longer *perpendicular B F* falleth.

45. The *Height* of any *Figure*, is a line drawn from the top of it, *perpendicular* to the baſe, as *C G* is the height or true breadth of the *Rhomboides B C E F*, it is alſo the height of the Triangle *B C D*.



What Proportion *is.*

46. *Proportion*, or *Reaſon*, is the *habitude* or *relation* that one *Magnitude* hath to another, according to *Quantity*: As if 7 be compar'd to 3, then the reſpect that the *quantity* of the *Magnitude* 7, has to the *quantity* of the *Magnitude* 3, is the *Reaſon* or *Habitude* of 7 to 3. In every *Ratio* obſerve, That that *Quantity* which hath reference, or is referr'd to another,

is called the *Antecedent* of the *Proportion*, and the *Quantity* to which it is referr'd is called the *Conſequent*; as in the *Compariſon* of 7 to 3, 7 is named the *Antecedent*, and 3 the *Conſequent*.

When Quantities *have a Reaſon or Proportion one to another.*

47. *Quantities* are ſaid to have a *Reaſon* or *Proportion* one to another, when being Multiplied they exceed one another. Now there is no *Line*, how ſhort ſoever, but if, multiplyed, may exceed a longer *Line* given, but if (any *Superficies* as) a *Square* and *Right Line* be propoſed, the *Line* cannot be ſo augmented in length, as that it can be ſaid to exceed the *Square*, becauſe if infinitely increaſed, it hath ſtill no *Breadth*. Hence therefore 'tis clear, all *Homogenious* Magnitudes *i. e. Magnitudes* of the ſame kind, have a *Proportion* or *Relation* one to another.

When Quantities *have the same Proportion.*

48. *Numbers* or *Quantities* are ſaid to have a *Reaſon* or *Proportion*, when each *conſequent* are like parts of their reſpective *Antecedents*, for no quantity can be ſaid to be *big* or *little*, but as it is compared to another. For if theſe four Numbers 9. 3. 6. 2. be propoſed, then if 3 is contained, the ſame Number of times in 9 as 2 is in 6, I ſay, they are *proportional*, i. e. they are *Quantities* in the ſame *Proportion*, for there is the ſame *reaſon* of 9 to 3 as of 6 to 2. *Hence Note*, that the *Quantity* of any *Ratio* is known by dividing the *Antecedent* by the *conſequent*, as the *quantity* of the *Ratio* of 7 to 3 is expreſſed thus $\frac{7}{3}$, or of the *Ratio* of a to b thus $\frac{a}{b}$, wherefore often for brevities ſake in the following *Theorems* the quantities of *Ratio*'s, are denoted after this manner: $\frac{a}{b} \sqsubset = \text{or} \sqsupset \frac{c}{d}$ *That is*, the *Ratio* or *Proportion* of a to b is greater, equal or leſs than the *Ratio* of c to d.

Of the Number of Terms in a Proportion.

49. *Proportion* consisteth of 3 *Terms* at the least, the second whereof doth supply the place of two, and therefore this that I now call *Proportion*, is more rightly termed *Proportionality*, or the similitude or likeness of *Proportions*; for as 4 to 6, so is 6 to 9, that is the *reason* of a first Magnitude compared to a second, must be like that of a third compared to a fourth. So that when there is but 3 *Terms*, the *consequent* of the first *Ratio* is taken for the *Antecedent* of the second, for as 9 : 6 :: 6 : 4. *Proportion* it self generally denoting no more than the *Ratio* betwixt two *Quantities*.

50. One *Reason* is said to be *greater* than another, when one of the *Antecedents* contains its *consequent*, more times than the other *Antecedent* doth his *consequent*, as if (in the comparison of these 4 *Quantities* *A*. *B*. *C*. *D*.) *A* contains *B* more times than *c* doth *D*, then there is a *greater reason* of *A* to *B* than of *c* to *D*.

51. *An aliquot part* is a lesser Number in respect of a greater, when it measures it exactly, as 2 is an *aliquot part* of 6, because it

it is contained just 3 times in it, and for the same *reason* 3 and 4 are aliquot parts of 12, for that they are precisely contained in it.

52. An *Aliquant part* is a lesser Number in respect of a grater, when it doth not measure it exactly, as 3 is an *aliquant part* of 7, because it is not contained precisely any Number of times in 7, there being above 2 times 3 in it, and yet not 3 times, also for the same *reason* 2 is an *aliquant* part of 9.

53. When the *Antecedent* is *double*, *treble*, *Quadruple*, *&c.* of the *consequent*, 'tis termed *double*, *treble*, *Quadruple*, &c. *Reason*; and these Names do rather depend upon the *Antecedent* than the *Ratio* it self.

54. *Numbers* are in *continued* or *continual proportion*, when the intermediate Numbers betwixt the first and last are taken twice, *i. e.* as *Antecedent* to *Consequent*, so is the *consequent* taken as an *Antecedent*, to a fourth *Number* or *Quantity*, so 16 : 8 : 4 : 2 : are in *continued proportion*, for as 16 : 8 :: 8 : 4 :: 4 : 2 :

55. *Duplicate*, *Triplicate*, &c. *Reason*, is when the *Antecedent* is *Compounded* of two like *Reasons*, as let those four *Numbers* 324 : 108 : 36 : 12 : be in *continual proportion*, then the *Reason* of the first to the third

third is *Duplicate* to that of the firſt to the ſecond, alſo the *reaſon* of the firſt to the fourth is *triplicate* to that of the firſt to the ſecond. *Dr. Barrow* expreſſes it thus: $^{3}\frac{2\,4}{3\,6} = \frac{3\,2\,4}{1\,0\,8}$ twice, and $^{3}\frac{2\,4}{1\,2} = \frac{3\,2\,4}{1\,0\,8}$ thrice.

56. *Homologous ſides* are thoſe in any Figure which correſpond one to another, in *Numbers* or *Quantities*, the *Homologous terms* are the two *Antecedents*, and likewiſe the two *Conſequents*.

57. *Reciprocal Figures* are thoſe, whoſe parts may be ſo compared that the *Antecedent* of one *reaſon*, and the *conſequent* of the other, ſhall be found in the *ſame Figure*, *i. e.* when the *proportion* beginneth and endeth in the *ſame Figure*. In *Quantities* 'tis when it beginneth and endeth in the ſame *rank*, as if $\left.\begin{array}{r} a : b :: c : d : \\ \text{and } e : b :: c : f : \end{array}\right\}$ then reciprocally as $e : a :: d : f$.

Of the varieties of Proportion.

58. *Alternate Reaſon or Proportion*, is the comparing of *Antecedent* to *Antecedent*, and *Conſequent* to *Conſequent*, as if $A : B :: C : D$ then *alternately compar'd* it will be as $A : C :: B : D$.

59. *Inverse Reason* is when the *consequent* is taken as the *Antecedent* and so compared to the *Antecedent* taken as *Consequent*, as if $A : B :: C : D$: then *inversly* as $B : A :: D : C$.

60. *Compounded Reason*, is when the sum of the *Antecedent* and *Consequent* is *compar'd* to the *Consequent* it self, for if $A : B :: C : D$ then by *composition* of *Reason* it will be as $A + B : B :: C + D : D$.

61. *Divided Ratio*, is when the *excess* wherein the *Antecedent* exceeds the *Consequent* is compared to the *Consequent*, as if $A : B :: C : D$ then by *Division* of *reason* it will be as $A - B : B :: C - D : D$

62. *Converse Reason* or *proportion* is the comparing the *Antecendent* to the excess, wherein the Antecedent exceeds the *Consequent*, as if $A : B :: C : D$ then by *conversion* 'twill be as $A : A - B :: C : C - D$

63. *Mixt Reason*, is the comparing the *sum* of the *Antecedent* and *Consequent*, to the *difference* of the *Antecedent* and *Consequent*, for if $A : B :: C : D$ then mixtly as $A + B : A - B :: C + D : C - D$

64. *Proportion* of *Equality*, is when there are taken more *magnitudes* than two in one *Order*, and also as many *magnitudes* in another *Order*, then by comparing two

to two, if there be the ſame *reaſon* of *A* to *B* as of *E* to *F*, alſo of *B* to *C*, as of *F* to *G*, and of *C* to *D*, as of *G* to *H*. I ſay, that there will be the ſame reaſon of *A* to *D*, as of *E* to *H*, or it is a *compariſon* of the *extream terms* in both *Ranks*.

A. B. C. D.
E. F. G. H.

65. A *right line* is ſaid to be *cut* in *Extream* and *Meon Proporiion*, when as the leſſer part of the ſaid line is to the greater, as the greater is to the whole line, *i. e.* a *Rectangle* made of the whole line and leſſer Segment is equal to the *ſquare* made on the greater Segment.

Some Terms explain'd.

A *Theo.* or *Theorem* is when ſomething is propoſed to be demonſtrated.

A *Prob.* or *Problem* is when ſomething is propoſed to be done.

A *Prop.* or *Propoſition* is uſed promiſcuouſly, *i. e.* either for a *Theorem* or *Problem.*

A *Def.* or *Definition* is the unfolding or *Explicating* of the *Nature* and *Affection* of a thing.

A

A *Post*. or *Postulate* is a grantable request, or such a *Demand* as with reason may not be denied.

An *Ax*. or *Axiom* is a principal in any Art, so evident that it needs nothing but the light of Reason, to demonstrate it.

A *Cor*. or *Corrollary* is a consequent truth, gained from some preceeding Demonstration.

A *Schol*. or *Scholion* is a short critical Exposition, gained from a former Demonstration, or a Corrolary wanting an Explication.

Hyp. or *Hypothesis* is when a thing is supposed or given so to be, as if it is said *a* is equal to *b* by *Hypothesis*, it is as much as to say, *a* is equal to *b* by Supposition.

Con. or *Construction* is the drawing of Lines, and framing of Figures, or a preparing the Proposition for a Demonstration.

Dem. or *Demonstration* is the proving a thing by Definitions and Axioms, and so from several Arguments drawing a Conclusion, that it has that affection which the proposition did assume it.

Lem. or *Lemma* is the Demonstration of some Premise, in order to shorten a following Demonstration.

Poſtulates, or grantable Demands.

1. It may not be denied, but that a line from any point to any point may be drawn.

2. That a line may be produced to what length you pleaſe.

3. That from a given Centre, a Circle may be deſcribed with any diſtance.

Axioms.

1. Things equal to the ſame third, or equal things, are alſo equal one to another.

2. If to equal things, you add the ſame or equal things, the wholes ſhall be equal.

3. If from equal things, the ſame or equal things be taken away, the Remainders ſhall be equal.

4. Things *double*, *treble*, *Quadruple*, &c, (or a *Half*, a *Third*, a *Quarter* &c.) of the ſame or equal things, are alſo equal one to another.

5. The whole is greater than any of its parts, but equal to all of them taken together.

6, Thoſe

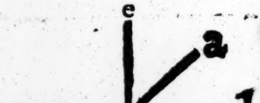
6. Thoſe things which fit one another, and agree in all their parts, are equal.

7. If one whole be double to another, and that which is taken away from the firſt, double to that which is taken away from the ſecond, the remainder of the firſt, ſhall be double to the remainder of the ſecond.

Geometrical Theorems.

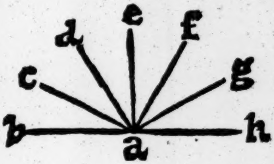
THEO. I.

IF *a straight Line* a b *fall upon another straight Line* c d, *it makes the Angles* a b c, a b d *on each side it, either both right, or both equal to two right.*



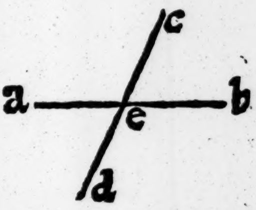
Demonstration. If the $\angle$ *a b d* $=$ *a b c* then (*per Def.* 33.) are they both right, if not, let *e b* be a perpendicular, then is the $\angle$ *a b d* + *a b e* equal to one right (*per def.* 33. and *ax.* 5) to which if you add the $\angle$ *e b c*, all the angles together will make up two right (*per ax.* 5) *which was to be demonstrated.*

Cor. 1. Hence it follows, that how many right lines ſoever as *c a*, *d a*, *f a*, *g a*, meet together at one point, and on the ſame ſide of any ſtraight line as *b h*, all the angles formed by them at that point, are only equal to two right (*per* ax. 5.)



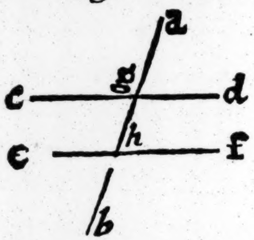
2. If to any ſtraight line, as *e a* and point therein *a* two right lines *b a*, *h a* be drawn on contrary ſides, making the Angle *b a e*, *e a h* equal to two right, then ſhall thoſe *lines* make one ſtraight line.

Schol. 1. From this *Theo.* 'tis evident, that if two right lines *a b*, *c d* cut through each other, the oppoſite Angles at the point of cutting are equal; For the $\angle$ *a e c* + *a e d* is equal to two right Angles (*per this Theo.*) Alſo by the ſame, the $\angle$ *a e d* + *d e b* is equal to two right Angles, therefore (*per. ax.* 3) the $\angle$ *a e c* = *d e b* *W. W. D.*



Cor. Hence all the Angles made about one point, are equal to four right.

Schol. 2. Hence alſo if a right line *a b* cut through two parrallel right lines *c d*, *e f* 'tis manifeſt 'twill make the alternate Angles equal, and the Exterior equal to the Interior, *i. e.* the $\angle f b g = c g b$ and the $\angle a g d = a b f$ for two parrallel lines are but the bounds or outſides of one broad line, and therefore from the foregoing demonſtrations, 'tis plaine that the $\angle f b g = c g b$ and the angle $a g d = a b f$ *W. W. D.*



Uſe. 'This *Theo.* is of great uſe, both in 'plain and ſpherical Trigonometry, for 'when one of the Angles made by the 'falling of a perpendicular is known, the 'other is alſo known: For Example, if 'the angle *a g d* in the laſt figure be 60 de-'grees, the Angle *a g c* is 120 degrees, the 'complement thereof to 180 degrees, and 'is found by ſubſtracting 60 degrees there-'from.

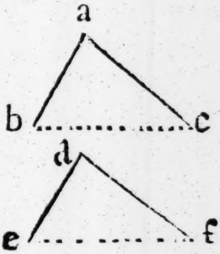
'Alſo from the laſt *Cor.* may be determi-'ned what polligons may be joined to-'gether for paving, and by it we find that 'either 6 Triangles, 4 Squares or 3 Hexa-'gons will perform the buſineſs; for the '6 Angles of an equilatteral Triangle, the

'4 of

'4 of a Square, and 3 of a Hexagon are
'equal to 4 right.

THEO. II.

If two Triangles a b c, d e f *have two sides* a b, a c *of the one, equal to two sides of the other* d e, d f *respectively, and have also the included angle* a *of the one, equal to the included angle* d *of the other, then shall they have the rest of their parts respectively equal, and consequetly the two Triangles shall be equal the one to the other.*

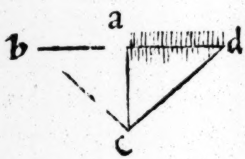


Dem. For lay the point *d* on the point *a*, and the right line *d e* on the right line *a b*, then shall the point *e* fall on the point *b*, for *d e*=*a b* (by *hyp.*) and the right line *d f* upon the right line *a c*, because (by *hyp.*) the angle *b a c*=*e d f*, also the point *f* will fall on the point *c*, for *d f*=*a c* (by *hyp.*) and therefore will the rest of the parts (*per ax.* 6.) be equal (*i. e.* the angle *a b c*=*d e f* and angle *a c b*=*e f d*, also the line *b c*=*e f*) and consequently the two Triangles. *W. W. D.*

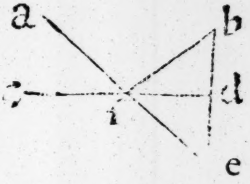
Schol. After the same manner it might be prov'd, that if either two Angles and a side

ſide comprehended, or all three ſides, be equal to the three reſpective ſides or parts of another △, then ſhall the two Triangles be equal one to another.

Uſe. ‘Hence may an inacceſſible diſtance ‘(ſuppoſe *a d*) be meaſured, for from *a* erect ‘the line *a c* perpendicular to the line *a d*, ‘then deſcribe a ſemicircle on the point *c*, ‘to meaſure the Angle *a c d*, and make *a c b* ‘equal thereto, then draw *b c* till it meet ‘with the line *d a* produced; now becauſe ‘in the two Triangles *b a c* and *d a c* there ‘is a common ſide namely *a c*, and two Angles in one, *viz.* *b a c* and *b c a* equal to ‘two angles *c a d* and *a c d* in the other, ‘therefore ſhall the reſt of the parts be equal, ‘and conſequently *b a* = *a d*, ſo that meaſuring the acceſſible line *b a* you have the ‘length of the inacceſſible line *d a*.



‘Alſo by this Theo. we are taught how ‘to ſtrike a Bowl at *Billiards*, ſo that by its ‘reflection it will hit any other Bowl propoſed.



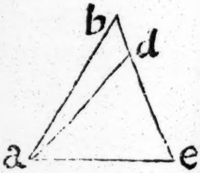
‘ Suppose one bowl at
‘ the point *a*, and the o-
‘ ther you would hit at
‘ *b*, and *c d* the *Billiard*
‘ *Table*, imagine thro’
‘ the point *b*, the line *b e*
‘ be drawn perpendicular to *c d*, and let *d e* =
‘ *d b*, and join the points *a. e.* then I say, that
‘ *f* is the point to which if the bowl at *a* be
‘ directed, it will by reflection carry it to *b*.
‘ For in the Triangles *b f d*, *e f d* there are
‘ two sides and an Angle respectively equal,
‘ and therefore by the precedeing *Theo.* the
‘ Angles *b f d*, *e f d* are equal; but by (*Schol.* 1
Theo. 1) the Angle *a f c* = *e f d*, and there-
‘ fore by (*ax.* 1) *a f c* = *b f d* that is the An-
‘ gle of incidence equal to the Angle of re-
‘ flection, and therefore the reflection will
‘ be by *a f b*.

‘ ‘ Hence it is easie to conceive, how Light
‘ may be transfer’d from one place to ano-
‘ ther, *viz.* by only placing several glasses,
‘ so that the Angle of Incidence may be e-
‘ qual to the Angle of Reflection, by which
‘ means the light of a Candle may be brought
‘ out of one Room into another.

THEO.

THEO. III.

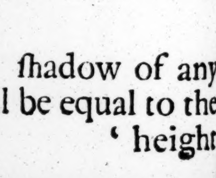
If a Triangle a b e *have two Angles in it equal, as if the angle* a $=$ e, *I say the sides* a b, b e *subtending those Angles shall be also equal.*



If you deny it, let one side be longer than the other, suppose *e b* $\sqsubset$ *a b*, then make *e d* $=$ *a b* by drawing the line *a d* (per *post.* 1.)

Dem. Now because in the two Triangles *d a e*, *b a e* there is a common side *a e*, and another as *e d* $=$ *b a* also the contained angle *d e a* $=$*b a e*, therefore (*per* Theo. 2.) shall the Triangle *a d e* $=$ *a b e* a part to the whole, *which is impossible.*

Cor. Hence if in any Triangle the 3 sides are equal, then are the three Angles also equal, *and contrary.*



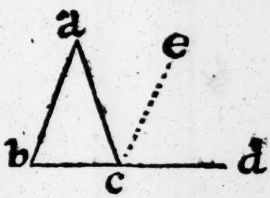
Use. ‘ By this Theo. the ‘ height of any *Object* from ‘ its shadow may be found, ‘ for do but wait till the ‘ Sun hath 45 Degrees of ‘ height, and at that instant ‘ measure the length of the shadow of any ‘ object, and that length will be equal to the ‘ height

'height of the object, for the angles *a* and *b*
'made by the Sun's ray *a b* with the object
'*b c*, and ground line *a c* are equal, and
'therefore (*per* this Theo.) the sides *a c*, *a b*
'are equal that subtend them.

THEO. IV.

In every right lin'd Triangle as a b c, *if one side* b c *is produced the outward* ∠ a c d *will be equal to the two interior and opposite angles at* a *and* b, *and the three interior angles of the triangle equal to two right.*

Con. From the point *c* draw *c e* parrallel to *a b*. *Dem.* then is the ∠ *e c d* = *a b c* (*per* Schol. 2. Theo. 1.) also by the same is the ∠ *e c a* = *c a b* and therefore (*per* ax. 2.) the two inward angles at *a* and *b* are equal to the whole exterior ∠ *a c d* W. W. D.



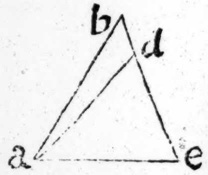
Secondly, If to the ∠ *a c d* you add the ∠ *a c b* it is evident (*per* theo. 1. and ax. 1.) that the 3 angles are equal to 2 right W. W. D.

Cor. 1. Hence it follows, that if one side of a triangle is produced, the outward ∠ formed thereby is greater than either of the inward opposite angles of the triangle.

2. That

THEO. III.

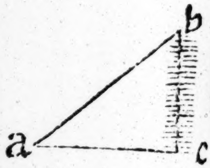
If a Triangle a b e *have two Angles in it equal, as if the angle* a = e, *I say the sides* a b, b e *subtending those Angles shall be also equal.*



If you deny it, let one side be longer than the other, suppose *e b* ⊏ *a b*, then make *e d* = *a b* by drawing the line *a d* (per *post.* 1.)

Dem. Now because in the two Triangles *d a e*, *b a e* there is a common side *a e*, and another as *e d* = *b a* also the contained angle *d e a* = *b a e*, therefore (*per* Theo. 2.) shall the Triangle *a d e* = *a b e* a part to the whole, *which is impossible.*

Cor. Hence if in any Triangle the 3 sides are equal, then are the three Angles also equal, *and contrary.*



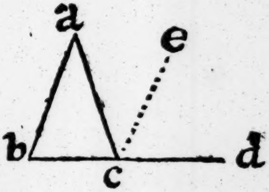
Use. 'By this Theo. the 'height of any *Object* from 'its shadow may be found, 'for do but wait till the 'Sun hath 45 Degrees of 'height, and at that instant 'measure the length of the shadow of any 'object, and that length will be equal to the 'height

‘ height of the object, for the angles *a* and *b*
‘ made by the Sun’s ray *a b* with the object
‘ *b c*, and ground line *a c* are equal, and
‘ therefore (*per* this Theo.) the ſides *a c*, *a b*
‘ are equal that ſubtend them.

THEO. IV.

In every right lin’d Triangle as a b c, *if one ſide* b c *is produced the outward* ∠ a c d *will be equal to the two interior and oppoſite angles at* a *and* b, *and the three interior angles of the triangle equal to two right.*

Con. From the point *c* draw *c e* parrallel to *a b*. *Dem.* then is the ∠ *e c d* = *a b c* (*per* Schol. 2. Theo. 1.) alſo by the ſame is the ∠ *e c a* = *c a b* and therefore (*per* ax. 2.) the two inward angles at *a* and *b* are equal to the whole exterior ∠ *a c d* W. W. D.



Secondly, If to the ∠ *a c d* you add the ∠ *a c b* it is evident (*per* theo. 1. and ax. 1.) that the 3 angles are equal to 2 right W. W. D.

Cor. 1. Hence it follows, that if one ſide of a triangle is produced, the outward ∠ formed thereby is greater than either of the inward oppoſite angles of the triangle.

2. That

2. That if one angle in a triangle is right, the other two taken together are equal to a right, but ſeverally acute.

3. That if one triangle hath two angles (ſeverally or together)equal to two angles of another triangle (ſeverally or together) then ſhall the remaining angle of the one, be equal to the remaining angle of the other ſo alſo if two triangles, have one angle of the one equal to one angle of the other, then is the ſum of the remaining angles in the one equal to the ſum of the remaining angles in the other.

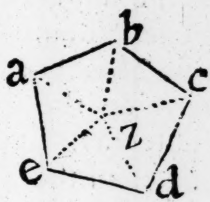
4. All the angles of an Equilatteral triangle, and the two angles at the baſe of an Iſoſceles triangle are acute.

5. That one angle of an Equilatteral triangle is $\frac{2}{3}$ of one right angle, for $\frac{1}{3}$ of two right angles is equal to $\frac{2}{3}$ of one.

Scol. 1. By help of this *Theo*, you may know how many right angles there are in any right lined figure. For all the angles of a right lined figure, do together make twice as many right angles wanting 4, as there are ſides of the figure.

Con.

Con. From any point within the figure, as *z*, let right lines be drawn to all the angles thereof, which lines will reſolve the ſaid figure into triangles, whoſe number will be equal to the ſides of the figure. Now whereas every triangle gives two right, all the triangles taken together will make up twice as many right angles as there are ſides, but the angles about the point *z* within the figure, are equal to 4 right (per *Cor. Schol.* 1. *Theo.* 1) therefore if from the angles of all the triangles, you take away the 4 right angles made about the ſaid point, the remaining angles will be twice as many wanting 4, as there are ſides of the figure.

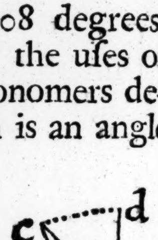


Cor. Hence all right lin'd figures of the ſame number of ſides, have the ſums of their angles equal.

Schol. 2. By this alſo the degrees of the angle of any regular polygon may be known for dividing the number of degrees in all the angles of a polygon, by the number of angles, the Quote will give you the degrees contained in one angle; thus procedeing you will find the angle of an Equilateral triangle 60 degrees, the angle of a Square

90, the angle of a Pentagon 108 degrees.

Uſe. ‘ Many and great are the uſes of
‘ this *Theo.* For ’tis by this Aſtronomers de-
‘ termine the Parallax (which is an angle
‘ ſubtended by the ſemidia-
‘ meter of the Earth.) As let
‘ *a* denote the Earths center,
‘ and from *b*, a point in the
‘ ſuperficies, let be taken by
‘ obſervation the angle *d b c*,
‘ which is the diſtance of the
‘ Star from the Zenith. Now



‘ tis plain, that if the Earth were tranſpa-
‘ rent, and the Star obſerved from *a*, its
‘ center, the $\angle\, c\,a\,d$ would be made, which
‘ is leſs than the angle *c b d*, by the $\angle\, a\,c\,b$,
‘ becauſe $c\,b\,d = c\,a\,d + a\,c\,b$ (per the
‘ precedeing *Theo.*) ſo that the $\angle\, a\,c\,b$ will
‘ be equal to the exceſs of the $\angle\, c\,b\,d$
‘ above $\angle\, c\,a\,d$; If therefore I can by any
‘ Artifice find how far diſtant from the *Ze-*
‘ *nith* the Star ought to appear at the center
‘ of the Earth, at the ſame punct of time,
‘ ’tis obſerved from the Superficies of the
‘ Earth, I may then find the quantity of the
‘ $\angle\, a\,c\,b$ which is the angle of parallax, and
‘ is ſubtended by the ſemidiameter of the
‘ Earth *b a*.

‘ Again, by the ſecond. *Schol.* Surveyors
‘ havh a method to prove whether (when

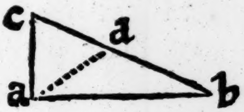
‘ in

'in ſurveying a **Field** by going round it) 'they have taken the angles right, for having obſerved them by an Inſtrument, 'find their ſum by adding them together; 'for Example, let the figure *a b c d e* (*Schol.* '1.) be a Field, whoſe angles had been 'taken. Turn the figure into triangles 'and find by the ſaid *Schol.* the number of 'right angles contained in the figure, then 'if the ſum of thoſe ſo found be equal to 'the ſum of thoſe which were taken, the 'work is right, otherwiſe not.

'*Laſtly*, By it may be conſtructed any 'regular polygon upon a given line, which 'is of great uſe in Fortification; for in 'the Ichnographick projection of any re-'gular Fort, the firſt thing to be done is 'to deſcribe a regular Polygon upon a gi-'ven ſide.

THEO. V.

The greateſt ſide c b *of any triangle, as* c a b *ſubtends the greateſt angle* a.



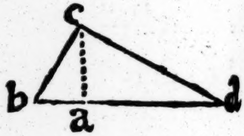
Con. Take from *c b*, *c d* = *c a* and join *a d* (*per* poſt. 1)

Dem. the ∠ *c d a* =

c a d (*per* Theo. 3.) but (*per* Theo. 4. Cor. 1.) the $\angle$ *c d a* $\sqsubset$ *c b a* i. e. the $\angle$ *c a d* $\sqsubset$ *c b a* (*per* ax. 5) In like manner it might be proved that the $\angle$ *c a b* is greater than the angle *c. W. W. D.*

Cor. Hence the greatest angle will always be subtended by the greatest side.

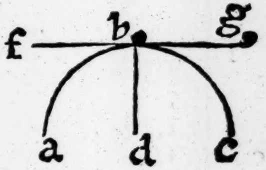
Use. 'The principal use of this *Theo.* is to 'prove that from one and the same point, 'and to the same straight line, there can be 'but one perpendicular drawn, and that 'this perpendicular is the shortest line or 'nearest distance from that point to the right 'line, to which it is a perpendicular; for 'Example, let *c a* be 'perpendicular to *b d*, 'I say then, that *c a* 'will be less than *c b*, 'because it subtends a 'lesser angle; after the 'same manner it might be proved, that '*c a* would be less than any other line 'drawn from *c* to any point in the line *b d*, 'except *a*.



'Also from hence it may be proved that 'a Bowl exactly round cannot rest upon a'ny plain parallel to the Horison, but up-

'on

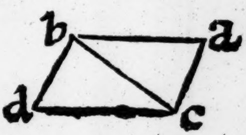
‘ on one certain point;
‘ for let *a b c* repre-
‘ ſent the ſurface of
‘ the Earth, and *d* its
‘ center, then let any
‘ plain ſuppoſe *f g* be
‘ a tangent to the
‘ Earths ſurface. Now becauſe it is the na-
‘ ture of all heavy bodies not to reſt when
‘ they may deſcend, and the bowl at *g* ha-
‘ ving nothing to hinder its nigher ap-
‘ proach to the center, muſt (by conſequence)
‘ continually deſcend till it comes to the
‘ point *b* where it reſteth, becauſe it is then
‘ nigheſt the center *d*. Hence it is alſo e-
‘ vident, that a liquid body muſt flow from
‘ *g* to *b*, and that its ſuperficies is round.



THEO. VI.

The oppoſite ſides a b, d c and b d, a c *in a* pgrm. *are equal, and the oppoſite* ∠s a, d, *and* a b d, a c d *are alſo equal, and the diameter* c b *biſects it.*

Dem. Becauſe (per *def.* 28.) *a b*, *c d* are parallel, therefore (*per Theo.* 1. *Schol.* 2.) the Angle *a b c* = *d c b*, alſo for the ſame reaſon



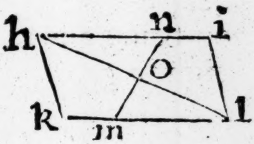
the angle $a\,c\,b = d\,b\,c$, and the ſide $b\,c$ common; therefore (per *Schol. Theo.* 2.) the Triangle $a\,b\,c$ will be equal to $b\,c\,d$, and conſequently the ſide $a\,b = d\,c$ and $a\,c = b\,d$. *W. W. D.*

Schol. Hence it might eaſily be proved, that if two equal and parallel Lines, ſuppoſe $b\,a$, $d\,c$ be joined together with two other right lines $a\,c$, $b\,d$ then are thoſe lines alſo equal and parallel.

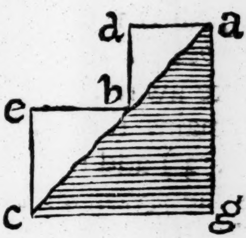
Uſe. 'Upon this *Theo.* is grounded the 'method of drawing all manner of parallel 'lines, as in the forgoing figure. Let it 'be required to draw through b, a paral-'lel to $d\,c$, join $b\,d$, and make $b\,a = d\,c$ 'and $a\,c = b\,d$ and the thing is done, for 'becauſe $b\,d$, $a\,c$ are equal and $b\,a = d\,c$, 'therefore (*per* this *Theo.*) $b\,d\,c\,a$ is a *pgrm.* 'and therefore hath its oppoſite ſides $b\,a$, '$d\,c$ parallel.

'Alſo by this *Theo.* is gotten a way of 'dividing any *pgrm.* into two equal parts, 'by a line that ſhall paſs through any point 'within the figure (which Surveyors have 'ſometimes occaſion to perform, in dividing 'a piece of Land into two equal parts, yet 'ſo as the Fence may paſs through a Pond 'or Lake which is in ſome part thereof.) As 'let it be required to divide the *pgrm.* $h\,i\,k\,l$ 'into

‘ into two equal parts, by a line that ſhall ‘ paſs through the point *m*, divide the di- ‘ ameter equally in *o*, and draw through ‘ *m*, and *o* the right line *o m*, which line ‘ ſhall divide the ſaid ‘ figure into two equal ‘ parts ; for the trian- ‘ gle *h n c*=*m l o* there ‘ being in either, two ‘ angles and a ſide ‘ comprehended by them ſeverally equal ‘ alſo the triangle *h k l*=*h i l*, *ergo* ſhall the ‘ *trapezium h n m k* be equal to the *trape-* ‘ *zium n i l m* (*per* ax. 2 and 3.) ſo that the ‘ *pgrm. h i k l* is biſected by the line *m n*, ‘ which was the thing to be done.



‘ Laſtly, by the *Schol.* of this *Theo.* may ‘ be meaſured the horizontal Lines and ‘ perpendicular Heighti of Mountains, which ‘ Lines are hidden by their ſolidity. For ‘ example, let the Mountain *a g c*, be to be ‘ meaſured ; take a ve- ‘ ry large ſquare as ‘ *a b d*, and placing ‘ one end thereof at *a* ‘ in ſuch ſort, that ‘ the other ſide *b d* ‘ may be perpendi- ‘ cular to the hori- ‘ zon, then meaſure the ſides *a d*, *b d*. Do ‘ the



'the like again at the point *b*, and measure the lines *b e*, *e c* then if you add the sides (*d a*, *e b*) which are parallel to the horizon together, the sum will give the length of the horizontal Line *c g*, and the sides (*b d*, *e c*) perpendicular to the horizon being added, gives the perpendicular height *a g*.

THEO. VII.

Parrallelograms b c d a, b c f e *standing on the same (or an equal) base* b c, *and betwixt the same or equidistant parallels* a f, b c, *are equal.*

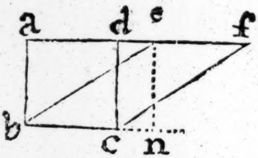
Dem. For *a b*=*d c* (*per Theo.* 6.) and *a d*=*b c* by the same, equal also to *e f* (*per hyp.*) add *d e* the common part, then is *a e*=*d f* (*per* ax. 2) Also the angle *e a b*=*f d c*, *ergo* the triangle *f d c*=*e a b* take away the common triangle *d g e*, there remains the *trapezium* *c g e f*=*a d b g* (*per* ax. 3) add the common triangle *b g c*, then is the *pgrm.* *a b c d*= *e b c f* (*per* ax. 2) *W. W. D.*

Cavalerius *his Demonstration of the former Proposion by the Mathod of* Indivisibles.

LET the former *pgrms. a b c d, e b c f* be proposed having the same Base *b c*, and being betwixt the same parallels *a f, b c*; divide the side *a b* in as many points as you please, Suppose in 1, 2, 3, and from those points draw Lines parallel to the Base, which produce to the outer side *c f* of the *pgrm. b c ef*: Now 'tis plain that there will be no more Lines in the one than there is in the other, neither will they differ in length, being all equal to the base *b c*, nor will they be closer in the one than in the other, for the lines are all parallel to the base, and consequently one to another, therefore will the two *pgrms.* be equal, because compounded and made up of like and equal parts.

Use. 'This *Theo.* persents us with the men-
' suration of any *pgrm.* (whether Square,
' Rectangle, Rhombus, or Rhomboides, for
' they all come under that denomination (by
' their *Def.* and *Theo.* 6,) as thus. Ima-
gine

'gine in a rectangled 'pgrm. *a b c d* the side '*a b* to be carried per-'pendicularly through 'the whole Line *b c*, 'or else *b c* though '*a b*, then shall this motion produce the 'Area of the *pgrm.* for it takes up the 'whole space *a d c b* passing through every 'phisical point therein.



'Hence a rectangle is said to be pro-'duced by the multiplication of two Lines-'As if *a b* be 5 and *b c* 3, draw 3 into 5: 'and 'twill produce 15 for the said rectan-'gle.

'This being supposed the Dimension of 'any other *pgrm.* as *e f b c* may be found, by 'multiplying the Base *b c* into the true 'height (which is a Line let fall perpen-'dicular to the Base from some point *e*, 'in the opposite side) as *e n*, the product 'of which giveth the Area of the *pgrm.* '*e f b c*.

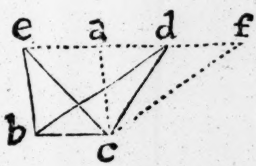
'Thus, if *e n* were 6 and *b c* 5 the Area 'of the Rhombodies or *pgrm.* will be 30. 'After the same manner is the Rhombus 'measured, by droping a perpendicular 'from an Angle to the side opposite, and 'multiplying any one side by the said per-'pendicular.

'The

'The ſquare is meaſured by *Multiplying* 'any one of its ſides into it ſelf, as if the 'ſide of a *Square* be 4, the Area of it will 'be 16.

HHEO. VIII.

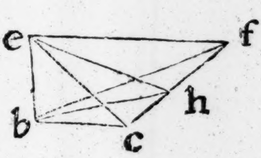
Triangles b c e, bcd *ſtanding on the ſame (or an equal) baſe* b c, *and betwixt the ſame or equidiſtant parallels* e f, b c *are equal*; *and contrary* i. e. *equal triangles* b c e, b c d, *on the ſame baſe (and ſide)* b c, *are alſo between the ſame parallels* e b c.



Con. Parallel to *b e* and *b d* draw *c a*, *c f* and compleat the *pgrms.* *b c a e*, *b c d f*.

Dem. Then (*per* Theo. 6.) *e c* biſects the *pgrm.* *b c a e*, as alſo *d c* the *pgrm.* *b d f c*, therefore the triangle $bce = \frac{1}{2}$ *pgrm.* *b c a e* $= \frac{1}{2}$ *pgrm.* *b d f c* (per *Theo.* 7.) $= \triangle bcd$ *W. W. D.*

Secondly, If *e f*, is not parallel to *b c*, draw *e h* parallel to it, and join *h b*, then is $\triangle bhc = \triangle bec$ (*per* this Theo.) =



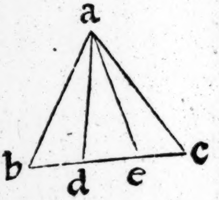
b f c

b f c (*per* hyp.) which is impoſſible, and againſt the 5 *axiom*.

Cor. Hence 'tis evident, that a parallelogram betwixt the ſame parallels, and on the ſame (or an equal) baſe with a trinangle, is double to it.

'*Uſe*. 'By this *Theo*. a practical way is 'found to divide a triangular Field into any 'Number of equal Parts;

'For Example, Divide the 'triangle *a b c* into 3 equal 'parts, triſect any ſide as '*b c* in the points *d* and *e*, 'and from the oppoſite '∠ *a* draw *a d*, *a e*, then 'are the triangles *b a d*, *d a e*, *e a c* equal '(*per* this *Theo*.) for they have the ſame 'height, and ſtand on equal baſes.

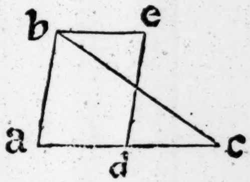


'This *Theo*. doth alſo preſent us with 'the reaſon of the Rule for meaſuring a tri-'angle; which Rule is to multiply half the 'baſe by the whole perpendicular, for 'ſince (by *Theo*. 7.) the whole baſe by the 'whole perpendicular produces the Area of 'a *pgrm*. which (per *Cor*. of this *Theo*.) is 'double to a triangle of the ſame (or e-'qual) baſe and height, therefore the 'whole of one line by the half of the other, 'will produce but half the Area of the *pgrm*.

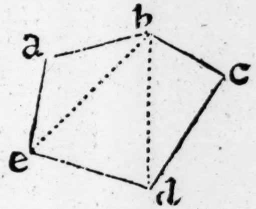
'which

'which must therefore by the said *Snhol.* be
'the Area of the triangle.

3. 'And from this Proposition, may easily be seen how a *pgrm.* may be made equal to any triangle, as let it be required to make a *pgrm.* equal to the triangle *a b c*, having bisected the base in *d*, and drawn a parallel thereto through the point *b*, take on the said parallel *b e* = *a d* and draw *d e*, then is the *pgrm b e a d* equal to the triangle *a b c*, as is manifest from the precedeing *Theo.*



4. 'It is also by this proposition, that the measuration both of the Circle, and also of all right lin'd Figures is performed. For Example, Let the Figure *a b c d e* be proposed to be measured, draw lines from Corer to Corner, and so reduce the Figure into triangles then measure each triangle severally (by leting fall a perpendicular, and measuring both it and the base by a Scale of equal parts) and collect all those Area's into one sum, so shall this total be the Area or content of the whole Figure.



5. If

'5. If it were required to measure a
'Circle, imagine the Circumference divi-
'ded into 100000 (or more) equal parts.
'Now I say, the difference betwixt one of
'those Arches and its subtense, will be so
'smal, that scarce any operation will re-
'quire greater exactness, but that the one
'may be taken as equal to the other, the
'difference being insensible: If therefore
'lines are conceived to be drawn from the
'Center, there will be 100000 triangles
'constituted, the sum of the Area's of
'which triangles will be equal to the Area
'of the Circle, and the sum of all the Bases
'equal to its Circumference, and therefore
'by the precedeing *Theo.* if the half sum of
'all the Bases (*i e.* half the Circumfe-
'rence) be multiplyed by the whole per-
'pendicular, which without sensible Errour
'may be taken for the Semi-diameter of the
'Circle, the Product will be the Area of all
'the triangles, and consequently the Area
'of the whole Circle.

'Hence 'tis evident how a triangle may
'be made equal to a given Circle. For
'having drawn the Semi-diameter *b a*, e-
'rect from *a* the perpendicular *a c* and

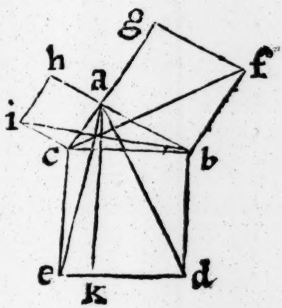
‘ make it equal to the circum-
‘ ference of the Circle, then
‘ join the points, *c*, *b*, by the
‘ right line *c b*, so shall this
‘ triangle *a b c* be equal to the
‘ Circle *a b d*, as may plainly
‘ appear from what before has
‘ been delivered.

THEO. IX.

In all right angled triangles as b a c, *the square of the Hypotenuse* b c *is equal to both the squares* h a c i, a b f g *of the other two sides added together.*

Let the figures *b c d e*, *f g b a*, *a b c i*, be the squares of the three sides *b c*, *b a*, *a c*, and let *a k* be drawn parallel to *c e*, as also the lines *b i*, *f c*, *a d*, *a e*.

Dem. Then because the angles *c b d*, *f b a* are equal (being both right) add to each the interjacent angle *a b c*, then is the ∠ *f b c* = *a b d* (*per ax.* 2) but *f b* = *d b*



(per

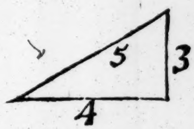
(per *def.* 24) and *b c* = *b d*, therefore the triangles *f b c* and *a b d* are equal (*per* Theo. 2,) but the *pgrm. b k* is double to the triangle *b a d*, (per *Cor. Theo.* 8) alſo the ſquare *b g* is double to the triangle *f b c*, for *g c* is a ſtraight line (per *Cor.* 2. *Theo.* 1.) therefore is the *pgrm. b k* equal to the ſquare *b g* (per *ax.* 4.) after the ſame manner it might be proved that the *pgrm. k c* is equal to the ſquare *a i*, Erg. (per *ax.* 2.) the whole ſquare *b e* of the ſide *b c* ſubtending the right angle, is equal to both the ſquares, *viz. b g*, *c h* made on the ſides *b a*, *a c* containing the right angle *W. W. D.*

Uſe. ‘ This *Theo.* is of ſuch great uſe, ‘ not only in ſeveral parts of the Mathematicks,. but alſo in many Mecanick Operations,, that 'tis ſaid *Pithagoras* the firſt finder thereof, in gratitude to the *Muſes* ſacrificed a Hundred Oxen to them, he ſuppoſing it (as by this it ſeems) beyond ‘ the power of bare humane Invention; neither was this his value or eſteem hereof ſo ‘ irrational, as to ſome perhaps it may appear, ‘ ſince 'tis the foundation of many conſideable practices in the Mathematicks.

‘ As *Firſt*, Trigonometry which cannot ‘ be performed without the aid and aſſiſtance hereof; for if in any right angled triangle, the Hypotenuſe and a leg be gi-

‘ ven,

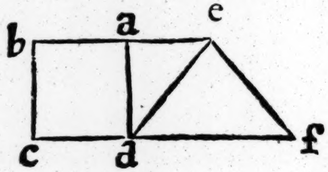
'ven, the other by this *Theo.* may be found, 'for having ſquared the hyp. which for ex- 'ample, I ſhall ſuppoſe 5, and likewiſe the 'given legg, which let be 3, then taking 9 'the ſquare thereof, from that of 5 which 'is 25, the remainder will be 16 the ſquare 'root of which is 4, the length of the o- 'ther ſide; thus likewiſe if the two leggs 'were given, the length 'of the hypotenuſe might 'be found, by ſquaring 'each legg, and adding 'the two products toge- 'ther, to wit, 9 and 16 the ſum of which 'is 25, and extracting the ſquare Root of 'this Sum, it will give 5 the length of the 'Hypotenuſe required.



' *Secondly*, 'Tis by help of this Prop, the 'Tables of natural Sines and Tangents are 'calculated, by which the parts of all tri- 'angles, both plain and ſpherical have their 'ſolution.

Thirdly, ' By this *Theo.* we not only can 'make a ſquare double, treble, Quadruple, '*&c.* to any one given, but alſo conſtruct 'a ſquare equal to any number of ſquares 'given; as firſt, to double the ſquare *b d* 'do thus, produce one ſide *b a* making *a e* 'equal to one of the ſides, then join the 'points *d e* and the thing is done, for the

'ſquare erected on the line *d a*, is equal to 'the ſquare made on the line *a e*, but both 'theſe are equal to the ſquare of *d e*, by the 'preceeding *Theo.* and therefore double to 'the ſquare *b d* which is one of them. But 'if it were required to make a ſquare equal to 'the ſum of the 2 ſquares made 'on the lines *d e* 'and *a e*, then



'from *e* erect the perpendicular *e f* equal to '*a e*, and join the points *f*, *d*, by the right 'line *f d*, ſo ſhall the ſquare made on the 'line *f d* be equal to the ſum of the ſquares 'made on the lines *d e* and *e f* by the pre-'ceeding *Theo.* Hence alſo may the ſub-'ſtraction of ſquares be performed, *i. e.* by 'it may be found the difference of any two 'ſquares, as if the ſquare made on the line '*f e* were to be taken from the ſquare of '*f d*, then by drawing a line equal to *f e*, 'and erecting from *e* a perpendicular *d e*, 'take between you compaſſes the length of 'the line *f d*, ſet one foot in *f*, and extend 'the other till it fall on the perpendicular 'at *d*, then join *f d* and it is done; for ſince 'the ſquare of the line *f d* is equal to the 'ſquares of the line *f e*, *e d*, therefore ſhall

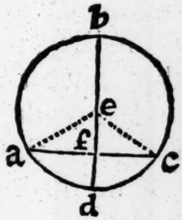
'the

‘ the ſquare of the line *d e* be equal to the
‘ difference of the ſquares *f d*, *f e*.

‘ Many other uſeful Practiſes mecanicks
‘ perform by this *Theo.* as the finding the
‘ length of ſtrings, Hyp Rafters, Scaling Lad-
‘ ders, and the like.

THEO. X.

If in a Circle e a c b, *a right line* b d *drawn through the Center biſect any other right line not drawn through the Center, it ſhall cut it at right angles, and if it cut it at right angles it ſhall biſect the ſame.*



Con. Draw from the Center *e* the right lines *e a*, *e c*.

Dem. Now becauſe *a f* = *f c* (*per* hyp.) and *e a* = *e c* (*per* dif. 16.) alſo the ſide *e f* common, therefore ſhall the angle *e f a* = *e f c* (*per* Theo. 2. and conſequently right (*per def.* 33.) *W. W. D.*

Again, becauſe the angles *e f a*, *e f c* are equal, and the ∠ *e a f* = *e c f* (*per Theo.* 3.) and the ſide *e f* common, therefore (*per Theo.* 2. and *Cor.* 5 of *Theo* 4.) ſhall *a f* = *f c* that is *a c* is biſected *W. W. D.*

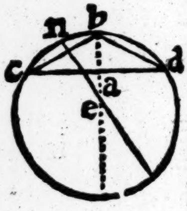
Cor. 1. Hence therefore the Center of a Circle is in that line, which biſects another at right angles.

 Cor.

Cor. 2. Hence in any Equilateral or Isosceles triangle, if a line drawn from the vertical angle bisect the base, that line is perpendicular to it and Contrary.

Use. ' By help of this *Theo.* the Center of ' a Circle may be found, for having applyed the right line *a c* in the Circle, *b a c* ' and bisected it with the perpendicular *b d*, ' which by the preceeding *Theo.* will pass ' through the Center, and is (*per* def. 12.) ' the Diameter; if therefore the said Diameter be divided into two equal parts in ' *e*, that point of Section shall be the Center.

' It also furnishes Artificers with a Geo-' metrical way of finding a Center thereby, ' so strike an Arch when ' they have the height *a b* ' and breadth *c d* given, ' for join the points *b c*, ' *b d* and produce *b a*, then ' bisect *c b* (or *d b*) with ' the perpendicular *n e*, pro-' ducing, it till it cut *b a* prolonged in *e*, ' which point is the Center that will sweep ' an Arch to the given Height and Breadth.



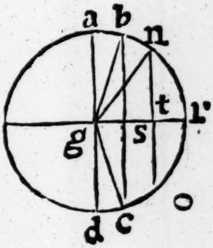
' *Lastly*, 'Tis used in Trigonometry, for ' by it is proved that the sine of an Arch is ' perpendicular to the Diameter; it also ' helps to demonstrate that the sides of a ' triangle have the same proportion as the ' sines of the opposite angles.

Theo.

THEO. XI.

In a Circle as g a c b *the greatest line* a d *is the Diameter, and the next greatest is that which is nearest the Center.*

Con. Draw the lines *g b*, *g c* and *g n*.



Dem. The Diameter *a d* = *g b* + *g c* (*per* def. 16.) but *g b* + *g c* ⊏ *b c* (*per* def. 2.) therefore *a d* ⊏ *b c W. W. D.*

Again, The sum of the squares of the lines *b s*, *g s*, are equal to the square of *g b*, (*per* Theo. 9.) so also are the squares of the lines *g t* and *n t*, for that they are equal to its equal, *viz.* *g n*. Now if you take away the square of the line *g s*, which is but part of *g t* from the square of *g b*, the square of *n t* will remain less than the square of *s b*, and therefore the line *n t* will remain less than *s b*, and consequently the whole line *n o* less than *b c*. *W. W. D.*

Use. 'This *Theo.* shews us that the Chord 'of an Arch greater or lesser than a Semi-'circle, is less than the Diameter.

'*Theodosius* also makes use of it to demon-'strate that in a Sphere, the lesser Circles are

 'more

'more remote from the Center than thoſe 'that are greater.

'It is alſo by this *Theo.* that *Ariſtotle* 'proves that the rowers in the middle of a 'Galley have greater ſtrength then thoſe 'that are either at the fore or hinder part 'thereof, for the ſides of the Galley being 'crooked, the owers of the middle part are 'longer, *i. e.* the *fulciment* or point of 'bearing comes nearer the middle of the 'Oar, and ſuppoſeing they pull with e-'qual ſtrength; he that ſits in the middle '(by this property) has his ſtrength en-'creaſed as the Doctrine of *Mecanicks* in 'the caſe of the *Ballance* and *Leaver* doth 'plainly exhibit and demonſtrate.

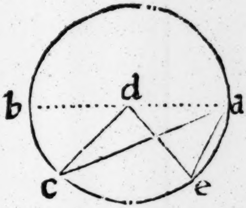
THEO. XII.

The angle b a c *at the Circumference of a Circle, is equal to half the angle* b d c *at the center ſtanding on the ſame, or equal arches* (b c) *of the Circumference.*

Dem. Caſe. 1. The external $\angle\, b\,d\,c =$ $d\,a\,c + d\,c\,a$ (*per* Theo. 4) alſo the line $d\,c = d\,a$ (*per* def. X6) therefore the $\angle\, d\,a\,c = d\,c\,a$ and conſequently the $\angle$ $b\,a\,c$ is equal to half the $\angle\, b\,d\,c$. *W. W, D.*

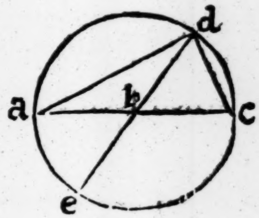
Caſe

Caſe. 2. The $\angle\, b\, d\, e$ is double to the $\angle\, b\, a\, e$ by the foregoing Caſe, and ſo alſo is the $\angle\, b\, d\, c$ double to $\angle\, b\, a\, c$, therefore (*per ax.* 8.) is the $\angle\, c\, a\, e = \frac{1}{2}\, c\, d\, e$ *W. W. D.*



Cor. Hence it follows that the angles ſubtended by (equal or) the ſame arch of the periphery are alſo equal, for that they are both double to the ſame thing, *viz.* the Angle at the Center.

2. That the angle in a Semicircle is right, which (though evident from the former *Theo.*) may be thus Demonſtrated. From *d* through the center draw the line *d e*, then is the $\angle\, a\, d\, e = \frac{1}{2}\, a\, b\, e$ and the $\angle\, e\, d\, c = \frac{1}{2}\, e\, b\, c$, but half the two angles *a b e* and *e b c* is a right angle (*per* Theo. 1) therefore the angle *a d c* is equal to one right angle *W. W. D.*

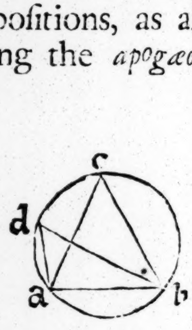


Schol. By this *Theo.* it may eaſily be demonſtrated, that angles made at the circumference contain half the quantity of the

Arch they ſtand on, for the arch *a e b* is the meaſure of the angle *a c b* (*per* def. 13) but the angle *a d b* = $\frac{1}{2}$ *a c b* therefore is the angle *a d b* equal too, but half the Arch *a b W. W. D*

Cor. Hence therefore it appears, that the angle made in a Segment greater than a Semicircle is leſs than a right angle, but an angle in a Segment leſs than a ſemicircle is greater than a right.

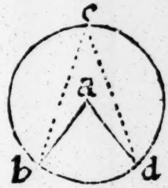
Uſe. ‘ This *Theo.* is uſed in demonſtrating ‘ ſome Trigonometrical propoſitions, as al‑ ‘ ſo in Aſtronomy for finding the *apogæon* ‘ and *excentricity* of the Sun.



‘ Alſo in Opticks it ſhews ‘ that the line *a b* will ap‑ ‘ pear of the ſame length ‘ when ſeen either from *d* ‘ or *c*, becauſe in both ‘ caſes you behold it under ‘ equal angles.

‘ Thirdly, a very eaſy and practical way ‘ is found to make an angle equal to half of ‘ any given angle ; as let it be required to

'make an angle equal to 'half the given angle *b a d*, 'on *a* as a center deſcribe 'with any radius the circle '*b c d* and from any point '(as *c*) in that part of the 'circumference, not inclu-



'ded in the given angle, draw the lines *c b* '*c d*, then is the angle *b a d* double to *b c d* '(*per* the preceeding *Theo.*) i. e. the angle '*b c d* $=\frac{1}{2}$ *b a d W. W. D.*

'Alſo the ſecond *Cor.* hereof ſhews Me-'chanicks how to try whether their ſquares 'be true, for having deſcribed a Semicircle '*a d b* they apply the head of their ſquare '*a d b* to the circumference, then laying 'one ſide as *d a* upon the extremity of the 'Diameter *a*, the other part or ſide *d b* will 'fall exactly on the other extream part of 'the Diameter *b*, if the ſquare be true. See 'the Figure of the ſaid *Cor.*

'Again, by the aforeſaid *Cor.* is ſhowne 'the manner of raiſing a perpendicular on 'the end of a given line; as alſo of letting 'fall one from a point aſſigned over the end 'of a given line.

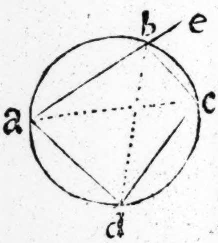
Theo.

THEO. XIII.

The opposite angles a + c *or* b + d *of every four sided Figure inscribed in a Circle, are equal to two right angles.*

Con. Draw from the opposite angles *a*, *c* and *b*, *d* the right lines *a c*, *b d*.

Dem. All the ∠s in the triangle *a b c* are equal to two right angles, but the ∠ *a d b* = *b c a* (per *Cor.* 1. *Theo.* 12.) also for the same reason is the angle *b d c* = *b a c* therefore the whole ∠ *a d c* = *b a c* + *b c a* equal to the complement of the ∠ *a b c* to two right angles, therefore the ∠ *a b c* + *a d c* is equal to two right angles. *W. W. D.*



Cor. Hence if one side of a Quadrilatteral figure inscribed in a Circle be produced, the outward ∠ *e b c* is equal to the internal angle which is opposite to the angle adjacent.

Use. ‘ This Proposition is helpful in demonstrating that the sines of the sides are in the same reason as are the sines of the opposite angles, *Ptolomy* also makes use of it to calculate the Table of *Chords*.

Theorems of Proportion.

THEO. XIV.

Magnitudes A B, *and* C. D *that have the ſame proportion to a third* E, F *have alſo the ſame one to another.*

Dem. For (*per* def. 6.) A will contain a third or any other aliquot part of B, as oft as E contains a third or any other the like aliquot part part of F. In like manner C contains the ſame aliquot part of D ſo oft as E doth the ſame of F, ſo that A contains an aliquot part of B, as oft as C doth the like aliquot part of D, and therefore I ſay, as A to B, ſo is C to D: Or it may be demonſtrated thus, $\frac{A}{B}=\frac{E}{F}=\frac{C}{D}$ therefore

A : B
3 : 6
C : D
4 : 8

E. F
5.10

(*per. ax.* 1) $\frac{A}{B}=\frac{C}{D}$, and therefore as A : B :: C. D. *W. W. D.*

Uſe. ‘This way of arguing is often made ‘uſe of, in ſeveral parts of the Mathema-‘ticks, perticularly in the ſolution of obli-‘que Spherical Triangles.

Theo.

THEO. XV.

If there be several proportional Magnitudes, viz. *if* A : B :: C : D :: E : F *then I say there will be the same reason of one Antecedent to its consequent, as of the sum of all the Antecedents, to all the consequents.*

A : B :: C : D :: E . F
6 : 9 :: 12 : 18 :: 24 . 36

Dem. Since A is to B, as C to D, A will containe an aliquot part of B, as oft as C contains the like aliquot part of D (*per* def. 48.) suppose the third: Now the third of B and the third of D is the third of the sum of B and D, and therefore the sum of A and C will containe the third of the sum of B and D as oft as A containes the third of B, therefore as A : B :: A + C : B + D and what is said of this will hold of many more, and therefore as A : B : : A + C + E : B + D + F. *W. W. D.*

Cor. Hence if like proportionals be added to like proportionals, the wholes shall be proportional.

THEO. XVI.

What reason any any two Magnitudes as A *and* B *have one to another, the same reason shall the*

the like aliquot parts (suppose the fourth) of those quantities have one to another, that is, as A *is to* B *so shall* E (*the fourth of* A) *be to* F, (*the fourth* B.)

Dem. Divide A into 4 equal parts as E, H, I, K, and B into 4 equal parts, as F, L, M, N; Now I say, that as E : F : : H : L : : I : M : : K : N for all the antecedents are equal each to other, so likewise are all the consequents. But (*per* Theo. 15.) as the sume of all the antecedents A, to the sum of all the consequents B, so is E to F. W. W. D.

$$A\ 24 \left\{\begin{array}{ll} E & 6 \\ H & 6 \\ I & 6 \\ K & 6 \end{array}\right. \qquad B\ 8 \left\{\begin{array}{ll} F & 2 \\ L & 2 \\ M & 2 \\ N & 2 \end{array}\right.$$

Cor. Hence 'tis also evident, that the double, treble, (or any other *Multiplex* whatsoever) of two quantities are in the same reason as the Quantities themselves.

Use. ' These two Theorems do many ' times happen in Mathematical Arguings, ' they are also necessary to demonstrate those ' that follow.

THEO. XVII.

If 4 *Magnitudes are proportional* (viz. *if as* A: B : : C: D) *they shall also be alternately so*, i. e. *as* A: C : : B : D

Dem.

Dem. If you deny it and ſay there is a greater reaſon of A to C than of B to D; A will contain an aliquot part of G, ſuppoſe the third more times than B containes the third of D (*per* Theo. 16. and def. 50) let A contain the third of C 7 times, and B the third of D 6 times,.

If A : B :: C : D
21 : 14 :: 9 : 6
then alternatly
A : C :: B :: D
21 : 9 :: 14 :: 6

having divided A into 7 parts, one of them will be third of C, having alſo divided B into 7 parts, the third of D ſhall not be contained in one of thoſe parts, wherefore the $\frac{3}{7}$ of A is equal to C, but the $\frac{3}{7}$ of B is not equal to D (*i. e.*) doth not contain D; now ſeeing there is the ſame reaſon of A to B as of C to D, there will alſo be the ſame reaſon of $\frac{3}{7}$ of A to $\frac{3}{7}$ of B, as of C to D, (*per*. Theo. 14. and 16.) And if the $\frac{3}{7}$ of A be equal to C, the $\frac{3}{7}$ of B, ſhall be equall to D, although the contrary follows if the former ſuppoſition be right, *viz*. that A hath a greater reaſon to C, than B to D, and thereſore as A to C ſo is B to D. W. W. D.

And here Note, That alternate reaſon hath place only, when the Quantities are of the ſame kind, for Heterogenous Quantities cannot be compared alternately.

Theo.

THEO. XVIII.

If there be 4 Magnitudes in the same proportion (viz. *if* A : B : : C : D) *they shall also be proportional when Inverted.*

Dem. For if A be as great in respect of B, as C is in respect of D, then on the contrary B is as little in respect of A as D is in respect of C, and therefore as B: A : : D: C. *W. W. D.* For the quantitie of proportion is more generally defined by *how much fold* rather than by *how many times* the consequent is contained in the antecedent.

If A : B :: C: D
6: 2 :: 9: 3
then inversly
B : A :: D : C
2: 6 :: 3 : 9

THEO. XIX.

If Magnitudes compounded be proportional, they shall be proportional when divided, that is if it be as A : D :: B : F *it shall by Division be as* A-D: D :: B-F : F.

If A : D :: B: F
16 : 4 :: 8: 2
then by division
A-D: D :: B-F-F
12 : 4 :: 6 :: 2

Dem. Since there is the same reason of A to D as of B to F, A will contain D (or an aliquot part thereof) so oft as B contains F

or

(or the like aliquot part.) Now if D be taken from A and E from B one or any other number of times, then (*per ax.* 3.) D is contained in the remainder A-D, as oft as F is in the remainder B-F, and therefore (*per* def. 48.) as A-D : D : : B-F : F. *W. W. D.*

THEO. XX.

If Magnitudes divided be proportional, they shall also be so compounded, as if A-D: D :: B-F : F *it shall also be as* A : D : : B : F.

Dem. This is but the converse of the former, for the part D is contained in A-D so oft as F is in B-F make each antecedent to contain his consequent once more by adding the respective consequents to them, then will the whole A and B contain their parts D and F a like number of times i. e. A so oft D as B doth F, and therefore (*per* def. 48.) as A : D :: B : F. *W. W. D.*

If A-D : D : : B-F : F
12 : 4 :: 6 : 2
then by composition
A : D :: B : F
16 : 4 :: 8 : 2

Use. ' The four foregoing proportions
' very frequently occur in several mathema-
' tical argumentations.

THEO. XXI.

If the whole be to the whole, as the part taken away is to the part taken away, the remainder shall be to the remainder, as the whole was to the whole, i. e. *If it be as* A : B :: D : F *it shall also be as* A-D : B-F :: A : B.

Dem. Because A : B :: D : F therefore alternately (*per Theo.* 17.) 'twill be as A : D :: B : F and by division 'twill be as A-D : D :: B-F : F. And again by alternation as A-D : B-F :: D : F : A : B (*per hyp.*) therefore (*per Theo.* 14.) A-D : B-F :: A : B. *W. W. D.*

If A : B :: D : F
10 : 6 :: 5 : 3
then 'twill be
A-D : B-F :: A : B

Cor. Hence if like Proportionals be taken from like Proportionals the remainder shall be Proportional.

Schol. Hence is *converse Ratio* demonstrated, for if A : B :: D : F then alternately as A : D :: B : F but by the preceeding *Theo.* as A : D :: A-B : D-F, and again by alternation as A : A-B :: D : D-F. *W. W. D.*

If A-B :: D : F
then by Conversion
as A : A-B :: D : D-F

After the like way of arguing may mixt *ratio* be demonstrated.

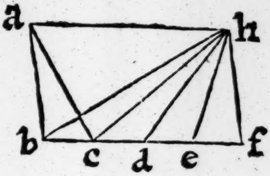
Use. ' Thls *Theo.* helps to demonſtrate the ' Rule of Fellowſhip, for inſtead of working by the Rule of Three for every particular Aſſociate, having done it for the ' reſt, to the laſt they aſſign the remainder ' of the gain: Suppoſing that if there be ' the ſame proportion of the whole ſume ' of all the principals to the whole gain, as ' of the principal of one aſſociate to his ' part of the gain, there will alſo be the ' ſame proportion of the principal that remains to the remainder of the gain.

' Some other ways of arguing there are ' which ſometimes are uſed, but theſe being the principal and moſt frequent, I ' omit them.

THEO. XXII.

Triangles b h c, b h d, b h e, b h f *whoſe Baſes are double, treble, quadruple,* &c. *to each other, ſtanding betwixt the ſame or equidiſtant parrallels, are alſo double, treble, quadruple to each other,* i. e. *they are in ſuch proportion to each other as are their baſes.*

Dem. Becauſe $bc = cd = de = ef$, and ab parrallel to bf, the triangles bac, bhc



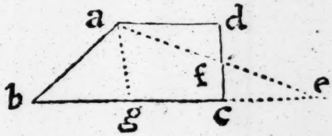
c h d, *d h e*, *e h f* are equal (*per Theo.* 8.) But *b d* = 2 *b c* therefore *b h d* = *b h c* + *c h d* = 2 *b h c* = 2 *b a c*. Again, *b e* = *b c* + *c d* + *d e* = 3 *b c*, therefore the triangle *b h e* = 3 *b h c* = 3 *b a c* (*per Theo.* 8. *ax.* 2.) alſo *b f* = *b c* + *c d* + *d e* + *e f* therefore *b h f* = 4 *b h c* = 4 *b a c* and therefore as the triangle *f h e* (or *b a c*) : *f h b* : : *f e* : *f b*. *W. W. D.*

Schol. In like manner it may be proved, that *pgrms.* of the ſame or equal height have ſuch proportion to each other as their baſes have, for (by the 16 *Theo.*) as the half is to the half, ſo is the whole to the whole.

Uſe. 'This *Theo.* ſhews how to cut off the 'third part of the *trapezium* (as *a b c d*) that 'hath two of its ſides (as *a d*, *b c*) parral- 'lel.

'Produce *b c* and make *c e* = *a d* then take '*b g* equal to the third of *b e* and draw *a g*, 'I ſay that the triangle *a b g* is one third of 'the *trapezium a b c d*.

Dem. The triangles *a d f*, *f c e* are equiangled becauſe of the parrallel Lines *a d*, *c e*, and vertical angle, there is



 alſo

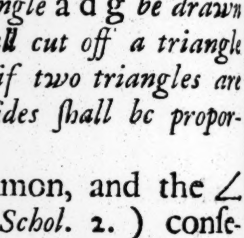
alſo in each an equal ſide, *viz.* *a d*, *c e*, (*per Con.*) thence they are equal to one another (*per Theo.* 2.) and conſequently the triangle *a b c* equal to the *trapezium a b c d*, but the triangle *a b c* is the third part of the triangle *a b e* by this *Theo.* and therefore the triangle *a b g* is a third part of the *trapezium a b c d*.

THEO. XXIII.

If to one ſide d g *of a triangle* a d g *be drawn a Line parralel as* c f *it ſhall cut off a triangle Simillar to the whole, and if two triangles are ſimillar, the correſponding ſides ſhall be proportional.*

Dem. The ∠ *a* is common, and the ∠ *a c f* = *a d g* (*per Theo. Schol.* 2.) conſequently the ∠ *c f a* = *d g a*, therefore (*per Def.* 35.) the triangles *a c f* and *a d g* are ſimillar. *W. W. D.*

Again, draw the Lines *c g*, *d f*, then the triangles *f c d*, *f c g* are equal, for that they have the ſame height, and a common baſe *f c*, but by the laſt as the triangle *a f c* : *f g c* :: *a f* : *f g*, alſo as *a c f* : *c d f* (= *f g c*) :: *a c* : *c d* and there-



fore

fore (by the 14 *Theo.*) $af : fg :: ac : cd$, but (*per* 17 *Theo.*) $af : ac :: fg : cd$. And (*per* 15 *Theo.*) $af : ac :: af + fg \, (= dg)$ $: ac + cd \, (= ad)$ and therefore (*per* 17. *Theo.*) as $af : ag :: ac : ad$. *W. W. D.*

Schol. Hence if the ſides of a △ be cut proportional, the right line that joins the points of Section, ſhall be parrallel to the remaining ſide of the △.

Uſe. 'This *Theo.* is of vaſt extent, and 'may almoſt paſs for an Axiom in all ſorts 'of meaſuring, it being the very foundation 'both of plain and ſpherical *Trigonometry*; for 'though ſpherical angles are formed by ar-'ches of Circles, yet muſt they all be re-'duced to plain triangles before their parts 'can be meaſured. Its alſo from this propo-'ſition they make and divide ſeveral Ma-'thematical Inſtruments, as ſinical Quadrant, 'Foreſtaff, Sector proportional Compaſſes, 'with other Geometrical Inſtruments. In 'ſhort, it is ſo univerſal, that moſt of the 'uſefulleſt parts of the Mathematicks are 'founded upon it. Its Uſes and Applicati-'ons being infinite.

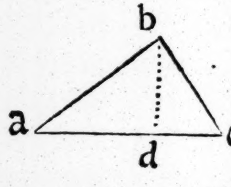
THEO. XXIV.

A Perpendicular let fall from the right angle of a right angled triangle as abc *divideth it*

 into

into two ſimillar triangles a d b,c d b, *which are alſo ſimillar to the whole.*

Dem. The ∠ *a b c* = *b d c* becauſe both right, and the ∠ *a* common, therefore the triangles *a b c*, *b d c* are ſimillar (*Theo.* 4. *Cor.* 3. *Def.* 35.) by the ſame argument *a b c*, *b d a* are ſimillar, and therefore alſo are the triangles *b d a*, *b d c* ſimillar. *W. W. D.*



Uſe. ' This *Theo.* is often uſed in ſolveing ' Geometrical Queſtions by Algebra, alſo ' by it and a ſquare, an inacceſſible diſtance ' as *d a* may be meaſured, for draw the per' pendicular *b d*, and ſet a ſquare at the ' point *b*, ſo that by looking over the ſides ' *b a*, *b c* you may ſee the points *a* and *c*. ' Now becauſe by this *Theo.* *c d* : *d b* : : *d b* : ' *d a*, if therefore you multiply *d b* by it' ſelf, and divide the product by *c d*, the ' quotient ſhall be *d a* the Line required.

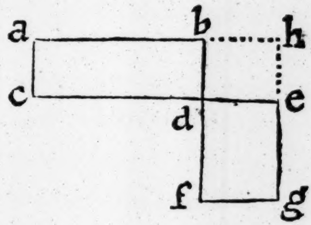
THEO. XXV.

Equal equiangled pgrms. *as* a d, d g, *have their ſides which are about the equal angles reciprocral, and equiangular* pgrms. *that have their ſides reciprocal are equal.*

Let

Let the ſides *c d*, *d e* about the equal angles make one right line, wherefore *f d*, *d b* (*per Theo.* 1. *Cor.* 2.) ſhall do the ſame, and let *a b*, *g e* be produced (*per Poſt.* 1.) till they meet in *h*.

Dem. Firſt therefore the *pgrms. ad*, *d g* being equal, have the ſame proportion to the *prgm. b e*, but the proportion of *a d* to *b e* is as *c d* to *d e*, and that of *d g* to *b e* is as *f d*, *b d* (*per Schol. Theo.*22.)

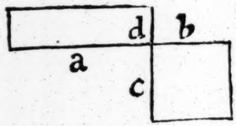


Ergo (*per Theo.* 14.) *c d* : *d e* : : *f d*, *d b*. *W. W. D.*

Secondly, the ſides of the *pgrm.* being reciprocal, there is the ſame proportion of *c d* to *d e* as of *f d* to *d b*, but as *a d* : *b e* : : *c d* : *d e* (*per Theo.* 22.) : : *f d* : *d b* (*per hyp.*) : : *d g* : *b e* (*per Theo.* 22.) therefore (*per Theo.*14.) *a d* is to *b e* as *d g* to *b e*, and therefore ſince $\frac{ad}{be} = \frac{dg}{be}$ it neceſſarily follows that *a d* and *d g* are equal. *W. W. D.*

THEO XXVI.

If a *be to* b *as* c *to* d *then the rectangle of* a *and* d *ſhall be equal to the rectangle of* b *and* c, i. e. *if four Lines be proportional, the rectangle of the Means ſhall be equal to that of the Extreams, and contra, if the rectangle of the Means be equal to that of the Extreams, the four Lines are proportional.*



Dem. The rectangles *a* and *b* have each an equal angle becauſe both right, and by *hyp.* $a : b :: c : d$, and therefore by the 2d. part of the laſt *Theo.* the rectangles are equal.

Secondly, If they are equal and equiangled their ſides will be reciprocal (by the firſt part of the laſt Theo.) that is, as $a : b :: c : d$. *W. W. D.*

Schol. In like manner it might be proved, that if three Lines are proportional, *viz.* *a*, *b*, *c*, the ſquare of the middle term *b* ſhall be equal to a rectangle of the firſt and third, *i. e.* $b\,b = a\,c$. *W. W. D.*

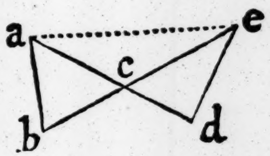
Uſe. 'By theſe two propoſitions is demonſtrated the principal Rules of common Arithmetick, *viz.* The Golden Rule, commonly called the *Rule of Three*, the Rules of Fellow-

Fellowſhip, Aligation, with others that depend on the Doctrine of proportion. For ſuppoſe it be required to find a fourth proportional to the three Numbers *a*, *b*, *c*. Then becauſe by this propoſition 9 x 6 = 3 multipled by another Number unknown. If then I divide 54 by 3 the quotient is 18, which if multiplied by 3 will give 54 = 9 x 6, and therefore if 3 give 9, 6 ſhall give 18 for the fourth proportional required.

a, *b*, *c*.
3, 9, 6.

THEO. XXVII.

Equal triangles a c b, e c d *that have in each an equal angle*, viz. a c b, e c d *have the ſides (* a c, b c *and* e c, d c *) which form the equal angles reciprocal; and contrary, if the angles are equal and ſides reciprocal, the triangles ſhall be equal.*



Let the ſides *a c*, *c d* which are about the equal angles be ſet in a ſtraight line, then *b c* is a ſtraight line *(per Theo.* 1. *Cor.* 2. join *a e*.

Dem. Firſt, as *a c* : *c d* : : △ *a c e* : △ *c d e (per Theo.* 22. *)* : : △ *e c a* : △ *c b a (* becauſe equal) *: : e c : b c (per Theo.* 22. *)* wherefore

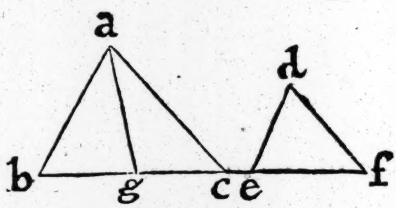
(per

(*per ax.* 1. and *def.* 48.) $\frac{ac}{cd} = \frac{ec}{bc}$ and therefore as $ac : cd :: ec : bc$. *W. W. D.*

Secondly, $\triangle ace : \triangle cde :: ac : cd$ (*per Theo.* 22.) $:: ec : bc$ (*per hyp.*) $:: \triangle eca : \triangle cba$ (*per Theo.* 22.) wherefore (*per ax.* 1. and *def.* 48. $\frac{\triangle ace}{\triangle cde} = \frac{\triangle eca}{\triangle cba}$, and therefore $\triangle cde = \triangle eba$. *W. W. D.*

THEO. XXVIII.

Simillar triangles a b c, d e f *are in duplicate proportion of the Homologous sides.*



Let $bc : ef :: ef : bg$ and let ag be drawn.

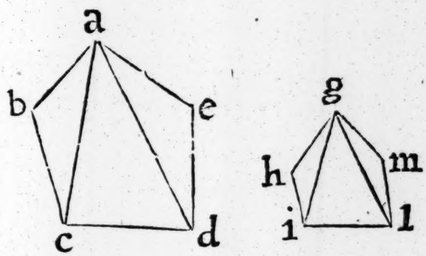
Dem. Because $ab : de :: bc : ef$ (*per Theo.* 23.) $:: ef : bg$ (*per hyp.*) and the angle $b = e$ therefore in the triangle $abg = def$ (*per Theo.* 26.) but $\triangle abc : \triangle abg :: bc : bg$ (*per Theo.* 22.) Now the proportion of bc to bg is duplicate to that of $bc : ef$,
def therefore $\triangle abc$ is in duplicate ratio to that of $bc : ef$, that is the triangle $\frac{abc}{def} = \frac{ef}{bc}$ twice. *W. W. D.*

Schol.

Schol. Hence alſo it might eaſily be proved, that ſimillar polligons, are in duplicate Ratio of their homologous ſides.

For ſimillar polligons *a b c d e*, *f g h i k* may be divided by the diagonals *a c*, *a d* and *g i*, *g l* into an equal number of ſimillar triangles, which triangles will be alſo like parts of their reſpective wholes.

Dem. The ∠ *b* = *b* (*per hyp.* (alſo ∠ *b a c* = *h g i* for the line *i g* that joins the points of termination (if △ *g h* be laid on △ *a b c*) is parrallel to



a c (*per Schol. Theo.* 23.) wherefore the triangles *a b c*, *g h i* are equiangled. In like manner it may be proved, that the triangles *a e d*, *g m l* are equiangled; and ſeeing that ∠ *b c a* = *h i g* and ∠ *a d e* = *g l m*, alſo the whole ∠ *b c d* = *h i l* (*per hyp.*) and ∠ *e d c* = *m l i*, the remaining ∠ *a c d* = *g i l* and ∠ *a d c* = *g l i* (*per ax.* 3.) and conſequently ∠ *c a d* = *i g l*, therefore the triangles *c a d*, *i g l* are ſimillar, and therefore, *&c.*

Now

Now becauſe the triangles *b a c*, *h g i* are like, therefore is $\frac{bac}{hgi} = \frac{ab}{gh}$ twice (*per* this *Theo.*) and for the like reaſon $\frac{cad}{igl} = \frac{cd}{il}$ twice, as alſo $\frac{aed}{gml} = \frac{ed}{ml}$ twice; and becauſe (*per def.* 48. and *Theo.* 17.) $ab : gh :: cd : il :: ed : ml$, i. e. $\frac{ab}{gh} = \frac{cd}{il} = \frac{ed}{ml}$ therefore (*per ax.* 1.) the triangle $\frac{bac}{hgi} = \frac{cad}{igl} = \frac{aed}{gml}$, *i. e.* $\triangle bac : \triangle hgi :: \triangle cad : \triangle igl :: \triangle aed : \triangle gml$, but (*per Theo.* 15.) as $bac : hgi :: bac + cad + aed : hgi + igl + gml$. So that $\frac{bac}{hgi} = \frac{bac + cad + aed}{hgi + igl + gml} = \frac{ab}{gh}$ twice, and therefore the whole figure $\frac{abcde}{fghil} = \frac{ab}{gh}$ twice, *i. e.* pol. *a b c d e* and pol. *f g h i l* are to one another, in a duplicate Ratio of their homologous ſides, *b c*, *h i*. *W. W. D.*

In like manner it may be proved, that Circles are in duplicate Ratio to that of their Diameters, for a Circle is onely made up of an infinite Number of triangles, as in the eighth *Theo:* was intimated.

Cor. Hence if 3 right lines are proportional, then as the firſt is to the third, ſo is a polligon made upon the firſt, to a polligon made

made upon the ſecond, like and a like deſcribed, or ſo is a polligon made upon the ſecond to a polligon like, and a like deſcribed on the third.

2. If the homologous ſides of like Figures be known, then will the proportion of the figures be evident, *viz.* By finding a third proportional.

Uſe. ‘ Hence is diſcovered a method of ‘ enlarging or diminiſhing any right lin'd ‘ figure in a given Ratio: As if you would ‘ make a Hexagon ſix times as big as another whoſe ſide is *a b*, then betwixt *a b* ‘ and 6 times *a b* find a mean proportional ‘ upon which deſcribe a Hexagon like the ‘ former, and it ſhall be Sextuple of the ‘ hexagon given.

‘ Alſo it corrects the Errour of thoſe ‘ who are apt to conclude that Polligons are ‘ in ſuch proportion to each other as their ‘ ſides: For if two Squares, two Polligone, *&c.* are propoſed, and the ſide of ‘ the firſt double to the ſide of the ſecond, ‘ then ſhall the firſt Figure be four times as ‘ big as the ſecond.

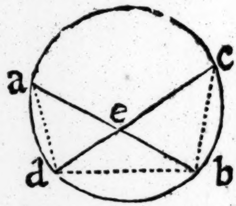
THEO.

THEO. XXIX.

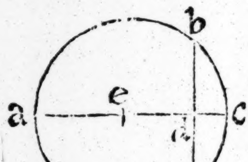
If two right lines a b, c d *intersect one another in a Circle, the rectangle made of the parts of the one, is equal to the rectangle made of the parts of the other,* i. e. a e x e b = d e x e c.

Join the points *d b*, *a d*, *c b*.

Dem. The ∠ *c e b* = *a c d* because vertical, alſo the ∠ *a* = *c* (*per Cor. Theo.* 12.) and conſequently the ∠ *a d e* = *c b e*, and therefore the triangles *a e d*, *c e b* are ſimillar, and (*per Theo.* 23.) have their ſides proportional, *i. e.* as *a e* : *e b* : : *e c*, *e d* and therefore (*per Theo.* 26.) *a e* x *e d* = *e b* x *e c*. *W. W. D.*



Schol. Hence is gotten a method of finding a mean proportional betwixt two given Lines, ſuppoſe *a d* and *d c*. In an infinite right Line, as *a c*, aſſume the point *d*, and on each ſide it (*per poſt.* 3.) cut off *d a* and *d c*, and ſuppoſing through *d* the right line *b h* was drawn



at right ∠s to *a c*, and that *a c* was bisected in *e*, describe with the distance *e a* or *e c*, the Circle *a b c*, then shall the line *d b*, or *d b* cut off by the periphery by the mean proportional required; it is also the side of a square equal to a rectangle made of the Lines *a d*, *d c*.

Dem. For $db = db$ (*per Theo.* 10.) and by this Theo. $bd \times d = ad \times dc$, i. e. $bd = adc$ and therefore as $ad : db :: db : dc$. *W. W. D.*

Use. 'This proposition is of great use in 'several parts of the Mathematicks; for by 'it may be found a third proportional to 'two given Lines, as also a fourth propor-'tional to three given Lines; and by it may 'be found a *pgrm.* having a side of any 'assigned length or breadth equal to any '*pgrm.* given; as also a square equal to a-'ny *pgrm.*

'And lastly, it gives Artificers the arith-'metical way of finding the Center, by 'which they may strike an arch that shall 'have an assigned height and breadth: As 'suppose I would find the Center that shall 'sweep the Arch *b c b*, whose height sup-'pose two foot, and breadth *b b* six foot. I 'first square half *b b* which is 3 it makes 9, 'which if divided by *d c*, or 2, will give in 'the quote $4\frac{1}{2}$ for the length of the Line

a d,

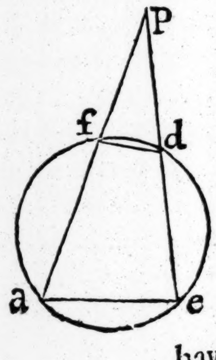
'*a d*, and this length added to *d* c, gives 6
'foot and an half for the length of the line
'*a c* the whole diameter, which if halued
'giveth 3 foot and $\frac{1}{4}$ diſtance from the height
'of the arch to the Center, which diſtance
'will ſtrike it to the given Dimenſions.

THEO. XXX.

If from a point p *taken without the Circle, you draw two right lines* p a, p e *to the oppoſite part of the periphery, the rectangle made of one whole line* p a, *with part of it* p f *cut off without the periphery, ſhall be equal to the rectangle made of the other whole Line* p e, *and that part of it* p d *cut off as before.*

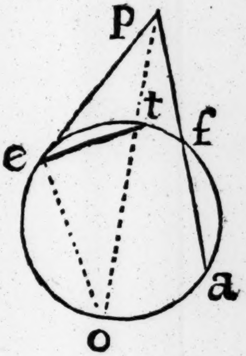
Join the points *a. e*, *f d*, with the right lines *a e*, *f d*.

Dem. The triangles *p a e*, *p d f* are ſimillar for the four ſided figure *a f d e* being inſcribed in a Circle, makes the ∠ *p f d* = *p e a*, they being both the Complement of the ſame angle, *viz. a f d* to a ſemicircle, and the angle *p* is common therefore (*per def.* and *Cor.* 3. *Theo.* 4.) are the triangle *p f d*, *p e a* ſimillar, and conſequently



have

have (*per* Theo. 23.) their correſponding ſides proportional, wherefore as $a\,p : e\,p :: d\,p : f\,p$ therefore (*per* Theo. 26.) $a\,p \times f\,p = e\,p \times d\,p$. *W. W. D.*



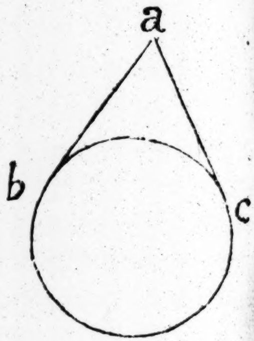
Schol. Hence it may eaſily be prov'd, that (if from a point p without a Circle, be drawn two right lines as $p\,e$, $p\,a$, ſo that one $p\,a$ may interſect the Circle ; and the other $p\,e$ touch it) the rectangle made of the one whole line $p\,a$ with that part of it $p\,f$ cut off without the periphery, ſhall be equal to the ſquare of the tangent line $p\,e$.

If the line $p\,a$ that cuts the circle doth not paſs through the center, then draw the prickt line $p\,o$ through it, and join the points $e.\,o$, $e.\,t$.

Dem. The $\angle\, e\,t\,o = e\,p\,o + pet$ (per *Theo.*4.) and the $\angle\, t\,e\,o$ is right, becauſe in a ſemicircle, and is therefore equal to the $\angle\, e\,t\,o + o$: Now the $\angle\, p\,t\,e + t\,e\,o =$ two right $\angle$s $+ o$ (*per* Theo.4.) $= \angle\, p\,e\,t + o + e\,p\,o + p\,e\,o$ (for $\angle\, t\,e\,o = \angle\, o + e\,p\,o + p\,e\,t$ (per *Theor.* 4.) take two rights from each there remains, $\angle\, p\,e\,t = o$;

 and

and the angle *e p o* is common, wherefore (*per* Cor. 3. Theo. 4.) the triangles *p e t*, *p o e* are ſimillar, and therefore (*per* Theo. 23.) have their ſides proportional, that is as *p t* : *p e* :: *p e* : *p o.* and therefore (*per* Theo. 30.) *p t* x *p o* = *p e* ſquare, but by this Theo. *p t* x *p o* = *p f* x *p a* therefore (*per ax.* 1.) *p f* x *p a* = *p e ſquare. W. W. D.*



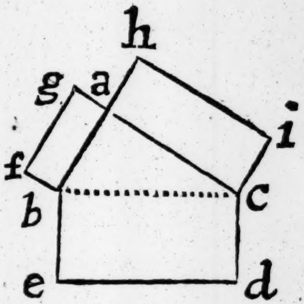
Cor. Hence it appears that two tangents as *a b*, *a c* drawn from the ſame point without the Circle are equal.

Uſe. 'The principal uſe of this Prop. is to demonſtrate the third and fourth axioms in plain Trigonometry.

THEO. XXXI.

In a right angled triangle b a c *any figure as* b d, *deſcribed on the ſide* b c *ſubtending the right angle, is equal to both the figures* b g, a i *deſcribed upon the ſides* b a, a c *containing the right angle, like and alike ſcituate to the former* b d.

Dem.



Dem. The figures *f a*, *a i*, *b d*, are like (*per hyp.*) and therefore (*per* the *Schol.* of *Theo.* 27) in a duplicate Ratio of their *homologous* ſides, alſo by the ſame *Theo.* the ſquares deſcribed on the ſides *b a*, *a c*, *b c* are in a duplicate Ratio of their ſides; Now ſeeing $ba_q + ac_q = bc_q$, therefore ſhall the figure $fa = ai = bd$ (*per* Theo. 14.) *W. W. D.*

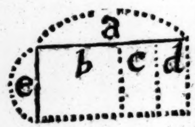
Schol. Hence may be learnt how to add or ſubſtract any like figures, *viz.* by uſing the ſame method that was followed in adding and ſubſtracting ſquares in *Theo.* 9.

Uſe. ‘ This *Theo.* exceeding uſefull, it be‘ing an univerſal application of the 9*th.* ‘ *Theo.* to all manner of figures.

EUCLID's Second BOOK,

Algebraically ſolv'd, and Geometrically Conſtructed and Demonſtrated.

PROP. I.



IF two right lines *a* and *e* be given, and one of them as *a* divided into as many parts as you pleaſe, *viz.* *b*, *c*, *d*, then the rectangle made of the two whole lines *a* and *e*, ſhall be equal to the ſeveral rectangles contained under the whole line *e*, and the ſegments or parts *b*, *c*, *d*.

That is, $ae = be + ce + de$

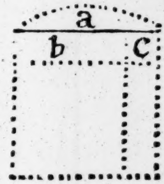
Solution Algebraically. *Let* $a = b + c + d$

Multiply each by e

it produceth $ea = eb + ec + ed$ *p. ax.* 4.

PROP. II.

If a right line *a* be cut any wise into two parts as *b* and *c*, then the rectangles comprehended under the whole line *a*, and each of the segments *b*, *c*, are equal to the square made of the whole line *a*.



That is, $aa = ab + ac$

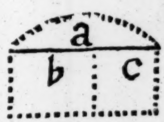
Sol. Alg. Let $a = b + c$

Mult. each by ——— a

it produceth $aa = ab + ac$ *p. ax.* 4.

PROP. III.

If a right line *a* be divided any how into two parts as *b* and *c*, then the rectangle comprehended under the whole line *a*, and one of the Segments *c* shall be equal to the rectangle under *b* and *c*, and the square of the said Segment *c*.



 That

That is, $ac = bc + cc$

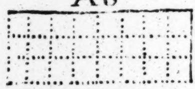
Sol. Alg. *Let* $a = b + c$

Mult. each by $\underline{\qquad\qquad c}$

it produceth $\underline{ac = bc + cc}$ *p. ax.* 4.

Use. ‘ This with the two foregoing pro-‘ positions serve to demonstrate the ordinary ‘ rule of *Multiplication* ; ‘ for Example, Let the ‘ number 8 represent ‘ the line *A*, and the ‘ number 3 the line *E*,

A 8



E 3

‘ if then (as before has been hinted) the ‘ line *E* be placed perpendicular to the line ‘ *A*, and in that posture be moved paral-‘ lel to it self, through every phisical point ‘ in the line *A*, it will produce the *rectangle* ‘ *A E* and that this motion representeth ‘ *Multiplication*, may be thus illustrated.

‘ Let there be assumed in the line *A* any ‘ number of *Mathematical points*, suppose 8 ‘ and in the line *E* let there be taken 3, now ‘ ’tis evident that in the *rectangle*, *A E* ‘ there shall be as many lines equal to *E*, as ‘ there are points in the line *A* : Multiplying ‘ therefore 8 by 3 (that is the number of ‘ points in the line *A*, by those in the line ‘ *E*) it will produce 24 for the number of ‘ *Mathematical points* in the whole *parrellelo-*

‘ *gram*

'*gram* *A E*, and this is the reason of that
'common notion in *Multiplication*, which is,
'that the Multiplicand must be put so of-
'ten into it self as there are Unites in the
'Multiplior, for the line *A* is increased in
'breadth so often as there were points ta-
'ken in the line *E*.

'But here Note, that by a *Mathematical*
'*point* must be understood any part or Quan-
'tity of a line divided at pleasure; for Ex-
'ample, If I suppose the line *A* to contain
'8 Yards 8 Feet or 8 Inches, then 1 Yard,
'1 Foot or 1 Inch I term a point, with-
'out considering that the same is composed
'of parts; for in measuring any line, I
'must make use of some line whose length
'is known, so likewise in the mensuration of
'Superficies, and Solids, *Use* must be made
'of a Superficies or Solid whose Dimenti-
'ons are known.

'This being premised, suppose the num-
'ber 43 were to be Multiplied by the num-
'ber 6, having divided 43 into two parts,
'*viz.* 40 and 3, I multiply 40 by 6 and it
'produceth 240, also I multiply 3 by 6 and
'it produceth 18, which two rectangles
'240 and 18 are equal to the whole rect-
'angle of 43 by 6, *i. e.* to 258, the reason of
'which equality is grounded on this *axiom*,
'*that if equal things be multiplied by the same*

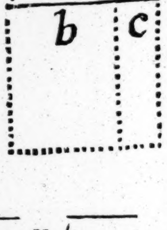
 '*or*

, or *equal things, the products ſhall be equal.*

$$\begin{aligned} &\textit{For } 43 = 40 + 3 \\ &\textit{Then Mult. each by} \underline{6} \\ &\textit{And it produceth } \underline{258 = 240 + 18} \quad \textit{p. ax. } 4. \end{aligned}$$

PROP. IV.

If a right line *a* be divided any wiſe into two parts, as *b* and *c*, then ſhall the ſquare of the whole line *a* be equal to both the ſquares of the Segments *b* and *c* and to a double rectangle of the ſaid Segments *b* and *c*.



$$\textit{That is, } a\,a = \overline{b + c} \times \overline{b + c}$$

$$\textit{Sol. Alg.} \text{ ——— } \textit{Let } a = b + c$$

$$\textit{Mult. each by an equal thing,} \text{viz. } a \text{ and } b + c$$

$$\textit{There is produced } a\,a = b\,b + b\,c + b\,c + c\,c. \quad \textit{p. ax. } 4.$$

$$\textit{or } a\,a = b\,b + 2bc + c\,c.$$

Cor. Hence it followeth, that the ſquare of half any line, is equal to one quarter of the ſquare of the whole line, for the ſquare of $\frac{1}{2}\,a$ is equal to $\frac{1}{4}\,a\,a$.

PROP. V.

If a ſtraight line a be cut into two equal parts each $\frac{1}{2}a$, and into unequal parts, *viz.* $\frac{1}{2}a+b$ and $\frac{1}{2}a-b$ then is the rectangle of the unequal parts added to the ſuqare of the differ. betwixt the equal and unequal parts, equal to the ſquare of the half line a.

That is, $\frac{1}{4}aa = \frac{1}{2}\overline{a+b} \times \frac{1}{2}\overline{a-b} + bb$

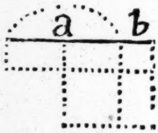
Sol. Alg. Let $\frac{1}{2}a+b$
be the greater part
Then ——— $\frac{1}{2}a-b$
is the leſſer part

and the rectangle of them is $\frac{1}{4}aa+\frac{1}{2}ab-\frac{1}{2}ab-bb$, *add to this* bb *it maketh* $\frac{1}{4}aa+\frac{1}{2}ab-\frac{1}{2}ab-bb+bb=\frac{1}{4}a$ *as is eaſily proved by expunging the parts that deſtroy each other in one ſide of the Equation.*

Uſe. ‘This, with the precedeing propoſition, is uſed in Demonſtrating the Method of finding the root of an adfected equation.

PROP. VI.

If a ſtraight line a be cut into two equal parts, each $\frac{1}{2}a$ and another right line b be put to the ſame directly in one ſtraight line, then is the rectangle of the whole and the line added taken as one line, and the line added, together with the ſquare of $\frac{1}{2}a$ equal to the ſame of $\frac{1}{2}a + b$ taken as one line.



That is, $\overline{a+b}\times b + \frac{1}{4}aa = \overline{\frac{1}{2}a + b}\times\overline{\frac{1}{2}a + b}$

Sol. Alg. $a + b$ $\qquad$ $\frac{1}{2}a + b$

Mult. by b $\qquad$ $\frac{1}{2}a + b$

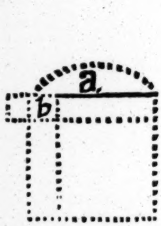
$\overline{ab + bb}$ *add to this* $\frac{1}{4}aa$ $\qquad$ $\frac{1}{4}aa + \frac{1}{2}$

$ab + \frac{1}{2}ab + bb \frac{1}{2}ab +$

It maketh $ab + bb + \frac{1}{4}aa$ ——— $\frac{1}{4}aa + ab + bb$

PROP. VII.

If a ſtraight line a be any wiſe cut into two parts as b and $a-b$, then the ſquare of the whole line added to the ſquare of one of the parts, is equal to two rect-



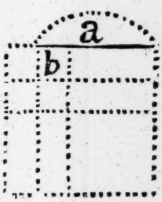
angles

angles under the whole line a and the said part b, with the square of the other part or Segment $a-b$.

That is, $aa + bb = 2ab + \overline{a-b} \times \overline{a-b}$.

$$
\begin{array}{l}
\textit{Sol. Alg.} \quad a - b \\
\qquad\qquad\quad \underline{a - b} \\
\qquad\qquad\quad aa - 2ab + bb \quad \textit{add to this } 2ab \\
\textit{It makes } aa - 2ab + 2ab + bb = aa + bb
\end{array}
$$

PROP. VIII.

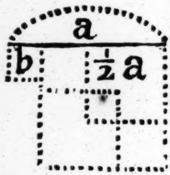


If a straight line a be cut any how into two parts as b and $a-b$, then four rectangles of the whole line and one of the parts, added to the square of the other part, will be equal to the square of the whole line, and the first Segment taken as one line.

That is, $4ab + \overline{a-b} \times \overline{a-b} = \overline{a+b} \times \overline{a+b}$

$$
\begin{array}{l}
\textit{Sol. Alg.} \quad a - b \\
\qquad\qquad\quad \underline{a - b} \\
aa - ab - ba + bb \quad \textit{add to this } 4ab \\
\qquad\qquad\qquad\qquad\quad a + b \\
\qquad\qquad\qquad\qquad\quad a + b \\
\textit{It makes } aa - 2ab + bb + 4ab = aa + 2ab + bb.
\end{array}
$$

PROP. IX.



If a ſtraight line a be cut into two equal parts each $\frac{1}{2}$ a and into unequal parts as b and $a-b$, then are the ſquares of the unequal parts b and $a-b$ double to the ſum of the ſquares of the half line a and the ſquare of the difference, *viz.* $\frac{1}{2}a-b$.

That is, $\frac{bb+\overline{a-b}\times\overline{a-b}}{2}=\frac{1}{4}aa+\overline{\frac{1}{2}a-}\times\overline{\frac{1}{2}a-b}$

$$\begin{array}{l} \textit{Sol. Alg.}\ a-b \\ \phantom{\textit{Sol. Alg.}\ }\underline{a-b} \\ \phantom{\textit{Sol. Alg.}\ }aa-2ab+bb \ \textit{add to this } bb. \end{array}$$

It maketh $\underline{aa-2ab+bb+bb}$

$$\begin{array}{r} \frac{1}{2}a-b \\ \underline{\frac{1}{2}a-b} \\ \frac{1}{4}aa-\frac{1}{2}ab-\frac{1}{2}ab+bb \\ \underline{\textit{add to this } \frac{1}{4}aa} \end{array}$$

It maketh $\frac{1}{4}aa-\frac{1}{2}ab-\frac{1}{2}ab+bb+\frac{1}{4}aa.$

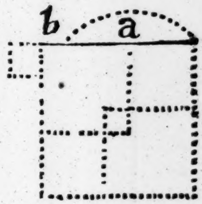
So that *$aa-2ab+bb+bb$ is double to $\frac{1}{4}aa+\frac{1}{4}aa-\frac{1}{2}ab-\frac{1}{2}ab+bb$.*

Uſe. ‘ This with the foregoing propoſitions are principally uſed in Algebra, for ‘ finding the root of a ſquare equal to a ‘ number

'number encreaſed more by a certain
'number of roots.

PROP. X.

If a ſtraight line *a* be cut into two equal parts each half *a* and another right line *b* be added to the ſame directly in one ſtraight line, then is the ſquare of the whole line, and the line added as one line double to the ſum of the ſquares of $\frac{1}{2}a$, and $\frac{1}{2}a+b$ taken as one line.



That is, $\overline{a+b}\times\overline{a+b}+bb=\overline{\tfrac{1}{2}a+b}\times\overline{\tfrac{1}{2}a+b}+\tfrac{1}{4}aa.$
2

Sol. Alg. $a+b$
$a+b$
$aa+2ab+bb$ *add to this* bb.
It maketh $aa+2ab+bb+bb$

$\tfrac{1}{2}a+b$
$\tfrac{1}{2}a+b$
$\tfrac{1}{4}aa+\tfrac{1}{2}ab+\tfrac{1}{2}ab+bb$ *add to this* $\tfrac{1}{4}aa$
It makes $\tfrac{1}{2}aa+ab+bb$ *that is the one half of* $aa+2ab+2bb$.

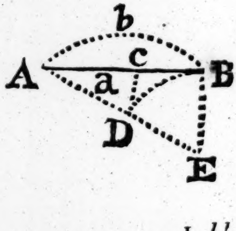
Prop.

PROP. XI.

To cut a line as b into two ſuch parts, that if upon the greater of thoſe parts be deſcribed a ſquare, it ſhall be equal to a rectangle made of the whole line and leſſer of thoſe parts.

That is, $aa = \overline{b-a} \times b$

Sol. Alg. Take the thing for done, and let the greater Segment be noted by a then the leſſer will be $b-a$, and the rectangle of the greater and leſſer $bb-ba$ which according to the propoſition muſt be equal to aa, that is $bb-ba=aa$, but by tranſpoſition $aa+ba=bb$; then by adding to either part of the Equation the ſquare of half the coefficient, it maketh $aa+ba+\frac{1}{4}bb=bb+\frac{1}{4}bb$, and then extracting the ſquare root of each we have $a+\frac{1}{2}b=\sqrt{:}bb+\frac{1}{4}bb$ or $a=\sqrt{\overline{bb+\frac{1}{4}bb}}-\frac{1}{2}b$.



Geometrically, let the given line be $AB=b$ erect from B, $BE=\frac{1}{2}AB$ perpendicular to it, and let be drawn AE, then (*per Theo.* 9.) is $AE=\sqrt{:}bb+$

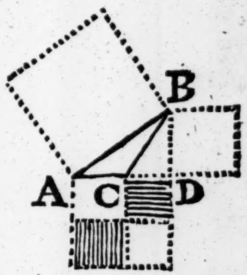
$\frac{1}{4}bb$,

$\frac{1}{4}bb$, on the center *E* with the diſtance *E B* $=\frac{1}{2}b$ deſcribe the Arch *B D*, this cuts off from the line *A E* the part *E D* $=\frac{1}{2}b$ and then the remaining part *A D* is equal to $\sqrt{bb+\frac{1}{4}bb}-\frac{1}{2}b=a$ and therefore *AC* (=*A D*) is the Segment ſought.

Uſe. ‘ This propoſition is requiſite for ‘ the inſcription of ſome of the regular bo- ‘ dies in a Sphere. It is alſo neceſſary to ‘ inſcribe a pentagone in a Circle, as alſo a ‘ pendecagone, and many other neceſſary ‘ practiſes it helpeth to perform. Father ‘ *Lucas* of St. *Sepulchres* compoſed a whole ‘ Book of a line cut in this manner.

PROP. XII.

In an obtuſe angled triangle *A C B* the ſquare of the ſide *A B* ſubtending the obtuſe angle, is greater than the ſum of the ſquares of the ſides *C A*, *C B*, (containing the ſaid obtuſe angle) by a double rectangle made of the ſide *C A*, and that part *C D* betwixt the ſaid obtuſe angle and the point *D* when the perpendicular *B D* falleth.



Uſe.

That is, $ABq = ACq$ x $CBq + 2AC + CD$.

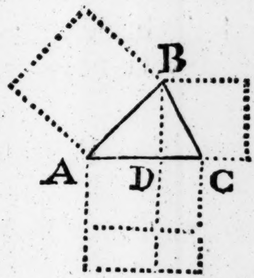
Dem. By *Theo.* 9. $ABq = BDq + AC9 + 2ACD + CDq$ and by the ſame $BCq = BDq + CDq$ therefore (*per ax.* 2) $ABq = BCq + ACq + 2ACD$. *W. W. D.*

Uſe. 'By this propoſition the Area of a 'triangle may be found by having its 3 ſides 'given, as let *A B* be 40 foot, *A C* 26, and '*B C* 22. Then I ſay, the ſquare of *A B* 'will be 1600, the ſquare of *A C* 676, and 'the ſquare of *B C* 484, the ſum of the two 'latter ſquares 1160, which being taken 'from 1600 leaves 440 for the two rectan-'gles under *A C*, *C D*; the one half of 'which is 220, for one of theſe rectangles 'which divided by *A C* 26, will give $8\frac{6}{13}$ 'for the line *C D*, whoſe ſquare is $71\frac{101}{169}$ 'which being ſubſtracted from the ſquare of '*B C* 484 there remains the ſquare *B D* 412 '$\frac{68}{169}$ the root of which is 20. 3 *prope*, for 'the ſide *B D*, which if multiplied by the 'half of *A C* which is 13, there will be 'produced 131. 95 for the Area of the tri-'angle *A B C*.

Prop.

PROP. XIII.

In an acute angled triangle *A B C*, the ſquare of the ſide *BC* ſubtending the acute angle *A*, is leſs than the ſum of the ſquares of the other two ſides *A B*, *A C*, by a double rectangle made of the ſide *C A*, and the Segment *D A*, lying betwixt the ſaid acute angle, and the point *D*, where the perpendicular falleth.



That is, $BCq = BAq + CAq - 2CAD$
Dem. By *Theo.* 9. $BAq - DAq = BCq$ $(-DCq) - CAq + 2CAD - ADq$. (per *prop.*7.) *Ergo* $BAq = BCq - CAq + 2CAD$ ſo that (per. *ax.* 2.) $BAq + CAq = BCq + 2CAD$. *or* $BCq = BAq + CAq - 2CAD$. *W. W. D.*

Obſerve that the Letters are ſo ordered, as to agree with the figure of *Prop.* 12. in caſe there be an obtuſe angle in the triangle.

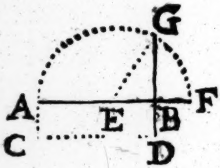
Note alſo in both Caſes, that the double rectangle of *DAC* added to the ſquare of *BC*, or its equal *BDq*+*DCq* makes the two ſquares of the lines *AB*, *AC*.

Uſe. ' This, as alſo the precedeing pro-
' poſition, are uſed in the ſolution of ſeve-
' ral algebraick Queſtions; they are like-
' wiſe very neceſſary in Trgonometry.

PROP. XIV.

To find the ſide of a ſquare equal to a given rectangle ABCD.

Geom. To *F* produce *AB*, makeing *BF*=*BD*, and biſect *AF* in *E*, on *E* with the radius *AE* or *EF* deſcribe the ſemicircle *AGF*, produce *DF* till it cut the Circle in *G*; then (per *Theo.* 29) is *GB* the ſide of a ſquare equal to the given rectangle.



Sol. Alg. Draw *EG* and let *AF*=*b* and *EB*=*a*, then is *AE*=*EF*=*EG*=$\frac{1}{2}b$, alſo *AB* =$\frac{1}{2}b+a$, and *BD*=*BF*=$\frac{1}{2}b-a$, and the rectangle $\overline{AD}$ (i. e $\overline{ABD}$) equal to $\frac{1}{2}b+a \times \frac{1}{2}b-a$ (i. e $\frac{1}{4}bb-aa$.) but *EGq*—*EBq* =*GBq*=$\frac{1}{4}bb-aa$, (per *Theo.* 9.) for the ſquare root of $\frac{1}{4}bb$ is $\frac{1}{2}b$=*EG*, and the ſquare root

root of *aa* is $a = EB$, therefore *GB* is the ſide of a ſquare equal to the rectangle *AD*. *W. W. D.*

Uſe. ‘ By this Propoſition any right ‘ lin’d figure whatever may be brought to ‘ a ſquare, for having reduced any irregu‘ lar figure into triangles, and then thoſe ‘ triangles into parrallelograms, as already ‘ hath been ſhewed, we may by this Pro‘ poſition reduce the *pgrms.* into ſquares, ‘ which are equal to them, and then (per ‘ *Theo.* 9) find a ſquare equal to the ſum ‘ of all thoſe ſquares.

The Doctrine *of* Proportion *in its various Changes and Combinations Algebraically Demonstrated.*

IF 4 Magnitudes are proportional (*viz.* if $A : a :: B : b$) they ſhall alſo be proportional in Alternation, Inverſion, Compoſition, Diviſion, Converſion, and Mixtion.

For if $A : a :: B : b$.

Then by
- Alternation $A : B :: a : b$
- Inverſion $a : A :: b : B$
- Compoſition $A + a : a :: B + b : b$ or $A + B : B :: a + b : b$
- Diviſion $A - a : a :: B - b : b$ or $A - B : B :: a - b : b$
- Converſion $A : A \pm a :: B : B \pm b$ or $A : A \pm B :: a : a \pm b$
- Mixtion $A + a : A - a :: B + b : B - b$ or as $A + B : A - B :: a + b : a - b$.

Lem. If 4 Quantities are proportional, the rectangle of the means, is equal to that of the Extreams.

Let

Let these 4 Quantities $8 : 4 :: 6 : 3$ be proposed, then I say that $8x3 = 6x4$. for $2x4 = 8$ and $3x2 = 6$. But $2x4x3 = 3x2x4$ (per *ax.* 4) therefore $8x3 = 6x4$. *W. W. D.*

This being granted, the foregoing varieties of proportion are easily proved.

For if $A : a :: B : b$ then by the preceding *Lem.* $Ab = aB$ and dividing both either by Bb, AB or Aa we have $\frac{Ab}{Bb} = \frac{aB}{Bb}$ and $\frac{Ab}{AB} = \frac{aB}{AB}$ and $\frac{Ab}{Aa} = \frac{aB}{Aa}$. If then the like terms in both Dividend and Divisor be expung'd, there remains $\frac{A}{B} = \frac{a}{b}$. and $\frac{b}{B} = \frac{a}{A}$. and $\frac{b}{a} = \frac{B}{A}$. and therefore as $A : B :: a : b$. which proves *alternate Ratio*, and by the other two, the inverses of both are prov'd, for as $a : A :: b : B$, also as $b : a :: B : A$.

Again, if $A : a :: B : b$ then is $\frac{A}{a} = \frac{B}{b}$ and then is $\frac{A}{a} \pm 1 = \frac{B}{b} \pm 1$. That is (reducing those mixt Quantities into Fractions) $\frac{A \pm a}{a} = \frac{B \pm b}{a}$, therefore $A \pm a : a :: B \pm b : b$ which

proves both *Compoſition* and *Diviſion*; the Alternations and Inverſions of which, follow from what was before proved.

Thirdly, Becauſe (as before was ſhewed) $\frac{a}{A}=\frac{b}{B}$ therefore $1\pm\frac{a}{A}=1\pm\frac{b}{B}$ that is (when reduced) $\frac{A\pm a}{A}=\frac{B\pm b}{B}$ and therefore $A\pm a : A :: B\pm b : B$, but by inverſion 'twill be as $A : A\pm a :: B : B\pm b$ which proves *Converſion*: The alternation of which doth alſo follow.

Fourthly, And becauſe (by Converſion) $A : A\pm a :: B : B\pm b$, and by the inverſe of Diviſion, $a : A-a :: b : B-b$, that is, becauſe $\frac{A}{A-a}=\frac{B}{B-b}$ and $\frac{a}{A-a}=\frac{b}{B-b}$ therefore (per *ax.* 2.) ſhall $\frac{A}{A-a}+\frac{a}{A-a}=\frac{b}{B-b}+\frac{b}{B-b}$ or $\frac{A+a}{A-a}=\frac{B+b}{B-b}$ and therefore it will be as $A+a : A-a :: B+b : B-b$ which proves *mixt Ratio*.

Fifthly, If never ſo many magnitudes are proportional, ſuppoſe $A : B :: C : D :: E : F$, then as one antecedent, is to one conſequent, ſo is the ſum of all the antecedents, to all the Conſequents.

For

For if $A : B :: C : D :: E : F$ then by the precedeing *Lemma* $AD = BC$ and $AF = BE$ but (per *ax.* 2) $AD + AF = BC + BE$ *i. e.* $A \times D+F = B \times C+E$, and turning this equality into a proportion, 'twill be as $A : B :: C+E : D+F$, but by alternation $A : C+E :: B : D+F$ and by Converſion $A : A+C+E :: B : B+D+F$; and again by alternation as $A : B :: A+C+E : B+D+F$. which was to be proved.

Sixthly, If A. B. C. D and *a. b. c. d* be propoſed, and $A : B$ as $a : b$, alſo $B : C$ as $b : c$, laſtly $C : D$ as $c : d$; then I ſay it will alſo be as $A : D :: a : d$.

For becauſe 48. $\frac{A}{B}=\frac{a}{b}$ and $\frac{B}{C}=\frac{b}{c}$ alſo $\frac{C}{D}=\frac{c}{d}$ therefore $\frac{A}{B} \times \frac{B}{C} \times \frac{C}{D}=\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d}$ (per *ax.* 4) that is $\frac{ABC}{BCD}=\frac{abc}{bcd}$ for the factors in each being equal, the products muſt needs be equal, and therefore as $A : D :: a : d$ which proves proportion of equality.

After the ſame manner may inordinate proportion be prov'd.

Laſtly, (That which has been left unprov'd by moſt Geometrick Writers, is by this way eaſily demonſtrated,) *viz.* If like proportionals be multiplied by like propor-

tionals, the products ſhall be proportional. As ſuppoſe $a : b :: c : d$ alſo $e : f :: g : h$; then I ſay 'twill hold as $ae : bf :: cg : dh$.

For if $a : b :: c : d$ then $\frac{a}{b} = \frac{c}{d}$ alſo if $e : f :: g : h$ then $\frac{e}{f} = \frac{g}{h}$ but $\frac{a}{b} \times \frac{e}{f} = \frac{c}{d} \times \frac{g}{h}$ (per *ax.* 4.) i. e. $\frac{ae}{bf} = \frac{cg}{dh}$ and therefore $ae :: bf :: cg :: dh$. *W. W. D.*

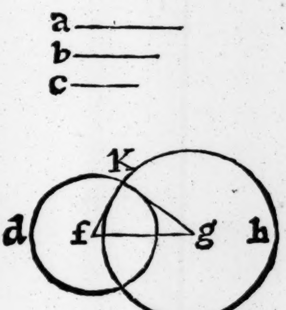
Geome-

Geometrical Problems.

PROB. I.

HOW *of* 3 *right lines* a, b, c, *to make a triangle* f g k, *of which it is necessary that any two of them taken together be longer than the third.*

On the infinite line *d h* take (per *post.* 3) *d f*, *f g*, *g h* equal to the given lines *c*, *a*, *b*, and from the centers *f* and *g* with the extents *f d*, *g h*, describe two Circles which will cut each other in *k*, then draw *f k*, *k g* and the triangle *f k g* shall be made, whose sides *f k*, *f g*, *g k* are equal (per *pos*. 3. def. 6.) to the 3 given lines, *a*, *b*, *c*. *W. W.* to be done.

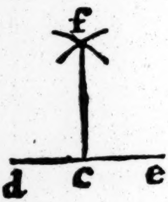


Schol. In like manner may be described either an Equilateral or Isosceles triangle, if

if ſo be that for the firſt, the 3 Circles be e-qual, and for the later two of them.

PROB. II.

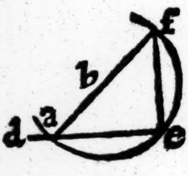
Upon a given right line d e, *and from any point therein as* c, *to erect a perpendicular* c f.



On either ſide the given point *c*, cut off (*per* poſt. 3.) *cd*=*ce* then with the di-ſtance *d e* (or any other not leſs than *d c*) and from the points *d*, *e*, deſcribe two Arches interſecting each o-ther in *f*, then join the points *f c* and it is done. The *Dem.* depends on the ſecond Theorem.

PROB. III.

If the perpendicular had been required to be raiſed in or near the end of the line d e, *ſuppoſe from the point* e, *then with any ſmall di-ſtance, ſet one foot of the Com. in* e *and extend the other (any where on that ſide the line* d e, *the perpendicular is to be raiſed) to* b, *on* b *as a Center, deſcribe with the ſpace*



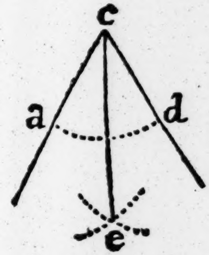
b e

b e *the ſemicircle* a e f, *drawing from* a (*the point where the Arch cuts the line* d e) *by the center* b, *the diameter* a b f, *then join the points* f, e, *with the right line* f e,*which is the perpendicular required. The* Dem. *depends upon* Cor. 2. Theo. 12. *where 'tis prov'd that an angle in a ſemicircle is right.*

PROB. IV.

To biſect or divide into two equal parts a given right lin'd angle c.

Set one point of the compaſſes in the angle *c*, and with the other cut off *c d* in the one leg, equal to *c a* in the other, on the points *d* and *a* with the ſame (or any other) diſtance deſcribe the prickt Arches interſecting each other in *e*; join the points *c*, *e*, with the right line *c e*, which performs what was required. The *Dem.* depends upon the ſecond Theorem.



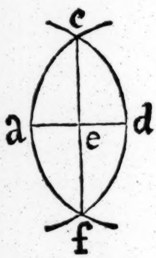
Cor. Hence it appears how an angle may be cut into 4. 8. 16. 32, *&c.* equal parts, *viz.* by a continual biſection of each part again. But the method of dividing angles into any number of equal parts by Rule and Compaſſes, is as yet unknown.

Prob.

PROP. V.

To divide a right line a d *into two equal parts.*

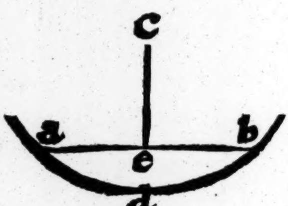
On the terms of the line *a d* or points *a* and *d*, deſcribe with any diſtance two Arches as *c a f*, *c d f* join the points *c*, *f*, with the right line *c f*, and it ſhall biſect the given line *a d* in *e*, for *a e* = *e d* as may eaſily be proved (*per* Theo. 2.) wherefore *a d* is biſected in *e*. *W. W. D.*



PROP. VI.

From a given point, as c *that is above, to let fall a perpendicular to a given line* a b.

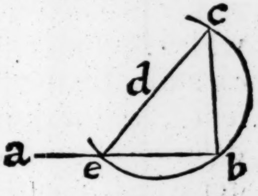
From *c* with any diſtance greater than *c e*, deſcribe an Arch of a Circle , cuting the given right line *a b* in the points *a* and *b*, then biſect *a b* in *e*, (*per* Prob 5.) and draw the



the line *c e*, and the thing is done, and is demonſtrated by *Theo.* 2. after the ſame manner as 4 of the precedeing Problems were.

PROP. VII.

To let a perpendicular fall from a given point c, *that is over the end of a given line* a b.



From any point as *e* near the middle of the given line *a b*, draw the line *c e*, which biſect in *d*, on *d* as a center with the diſtance *d c* or *d e*, deſcribe the ſemicircle *c b e* and join the points *c*, *b*, by the right line *c b*, and that is the perpendicular required.

It is demonſtrated from (*Theo.* 12.) as was the third Prob.

PROB. VIII.

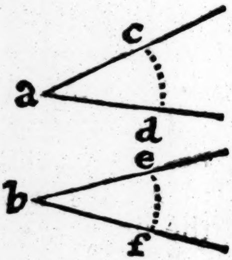
To draw a right line a b, *parallel to a right line given, as* c d, *and through a point aſſigned b.*

This caſe is univerſal, becauſe all others fall under it; and the Practice and Demonſtration is ſhown in the uſe of *Theo.* 6.

Prob.

PROP. IX.

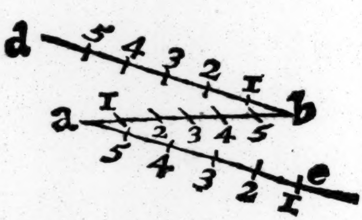
To make an angle b, *equal to a given angle* a



Draw the right line *b f*, then set one foot of the Compasses in the point *a*, and at any convenient distance, describe with the other the arch *c d*, also with the same extent set one foot in *b*, and describe the arch *e f*, after which take between your compass the distance *c d*, and set from *f* to *e* in that arch, and draw *b e* and the thing is done, and is easily demonstrated by the second *Theo.*

PROB. X.

To divide a line as a b, *into any number of equal parts.*



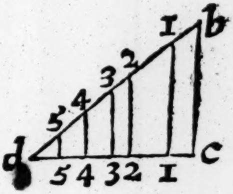
Suppose it be required to divide the line *a b* into 6 equal parts, draw from *b* the line *b d* making an

angle at pleaſure with the line *a b*, alſo from *a* draw the contrary way and ſide, the line *a e* making the ſame angle with *a b*, as the former line *b d* did, then with any convenient diſtance of your Compaſſes, run from *b* on the line *b d*, five equal Diviſions, for there muſt be always one leſs than the number you intend to divide the line into, do the like with the ſame ſpace upon the line *a e*,but the contrary way ; then laying your Ruler from 1 in one line,to 1 in the other line, you will cut the line *a b* in number 5, and the diſtance betwixt 5 and the point *b*, is the one ſixth of the line: Proceed thus, by laying the ruler over 2,2. 3,3, 4,4. 5,5. and it will divide the line *a b* into 6 equal parts as was required.

PROB. XI.

To divide a line d b *after the ſame manner as another given line* d c *is divided.*

Make with the two lines *d b*, *d c*, the angle *b d c* of any bigneſs, then join the points *b c*, and from the points 5. 4. 3. 2. 1 in the line *dc* draw the lines 1,1. 2,2. 3,3.4,4. 5,5.

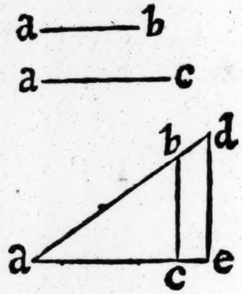


5,5. parallel to *b c* which will divide *d b* after the ſame manner as *d c* was before divided, the demonſtration of this and the precedeing Problem, is contained in the 23 *Theo.* where 'tis prov'd that if 1,1. 2,2. *&c.* be drawn parallel to *b c.* *d* 1, (in the line *d c*) will be to *d c*, as *d* 1 (in the line *d b*) to *d b*, and therefore, *&c.*

PROB. XII.

Two right lines a b, a c *being given, to find a third Proportional.*

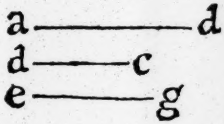
Make with the two given lines an angle at pleaſure, as *b a c*, then join *b c*, produce *a c* ſo that *a e* = *a b*, then from *e* parall. to *b c*, draw *e d*, and produce *a b* till it meet with *e d* in the point *d*, then

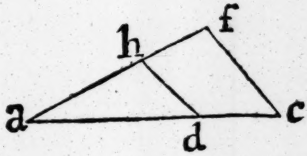


(*per.* Theo. 23) as *a c* : *a b* : : *a e* (*a b*): *a d.* So that the line *a d* is the third proportional required.

PROB. XIII.

Three right lines, a d, d c, e g, *being given to find a fourth proportional.*





In this Problem the fourth proportional ſhall be found decreaſing *i. e.* leſs than either of the 3 given lines, though in the precedeing Problem the third proportional was found increaſing that is greater. Make an angle at pleaſure as *f a c*, take upon *a c* the lines *a d*, *d c*, and upon *a f* the line *a h*=*e g*, then draw *h d*, and parallel to it, from the point *c* draw *c f* and its done; for as *a d* : *d c* :: *a h* (*e* g) : *h f* per *Theo.* 23.

PROB. XIV.

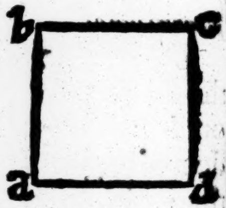
Two lines a d, d c *being given to find a mean proportional.*

See Scholion of *Theo.* 29.

PROB. XIV.

Upon a given right line as a d, *to make a Geometrical ſquare.*

Erect from *d* the perpendicular *d c*=*a d* on the points *c* and *a* deſcribe the two Arches, interſecting one another in *b*, then join *b c*, *b a* and the figure *a b c d* is a ſquare (per *def.* 30.) the demonſtration depends upon the ſecond and ſixth Theorems.



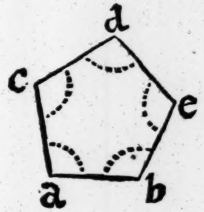
By the ſame Method may a rectangle or long ſquare be deſcribed, if the diſtance *c d* or *b a* be taken greater or leſſer than the other.

PROB. XV.

To deſcribe upon a given line, as a b *an equilateral Pentagone* a c d e b:

Find

Find the degrees contained in one angle (*per* Theo. 4. Schol. 2.) make (*per* Prob. 9.) an angle of 108 degrees with the given line *a b*, and the line *a c*, making *a c* = *a b*, then on the point *c* make another angle of 108 deg. drawing the line *c d* = *a c*; proceed thus with the rest of the angles and sides, till you come to the angle *e*, where you need but onely join the points *e b*, with the right line *e b* and the figure is compleated, for the angles *e* and *b* with the line *e b* are easily proved equal to the rest, wherefore the figure *a c d e b* is a regular pentagon per *def.* 31.

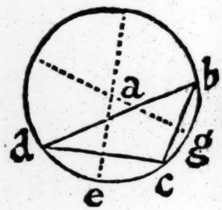


In like manner may any other regular polligon be described upon a given line, by laying down the angle as found by the 4*th*. *Theo*. *Schol*. 2. and so encloseing the figure with equal sides.

PROB. XVI.

To find a Center whereby to ſtrike a Circle, that ſhall paſs through any 3 aſſigned points, provided they are not placed in one ſtraight line.

Let the points be *d b c*, join them with the right lines *d b*, *d c*, *b c*, then biſect any two of thoſe 3 lines as *d c*, *b c*, and draw the right lines *e a*, *g a* from the points of biſection at right angles thereto, and where thoſe lines cut one another as in *a*, there is the center of the Circle, which (if deſcribed with the diſtance *a b*, *a c*, or *a d*) ſhall paſs through the given points *b*, *c*, *d*, as was required.



The Demonſtration depends on the 10*th.* *Theo.*

PROB. XVII.

To make a ſquare equal to a given rectangle.

This is eaſily deducible from the 29*th.* *Theo.* for the ſide of a ſquare equal to any

ny given rectangle, is onely a mean proportional between the longest and shortest side of it. See also the *14th*. Prop. of *Euclid's* second Book.

PROB. XVIII.

To make a square equal to any number of squares given.

See the Practice and Demonstration of it in the use of *Theo.* 9.

PROB. XX.

To divide a parallelogram into two equal parts, by a line that shall pass through any assigned point with the figure.

See the practice and Demonstration of this Problem in the use of the sixth *Theo.*

Many other Problems there are in Practical Geometry, which might sometimes be required, but I omit them, having only inserted those that are the most frequent, and generally required in the delineating of any Geometrick *Scheme* or Figure.

FINIS